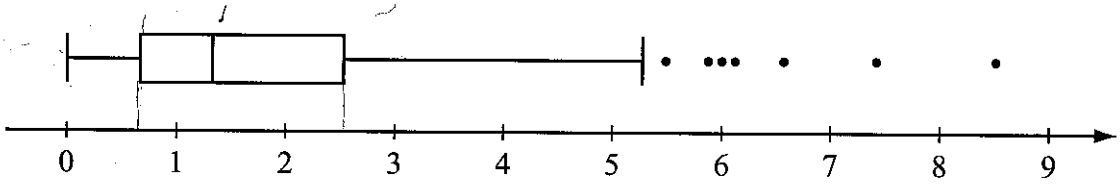
Sample Examination One

SECTION I
Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A company has recently given some pay raises. The distribution of the amounts by which the employees' salaries have been increased is illustrated by the boxplot below.



Pay Raise (thousand dollars)

Which of the following best describes the shape of the distribution and the interquartile range (IQR) of the salary increases?

(A) symmetrical; IQR is approximately $700
(B) positively skewed; IQR is approximately $700
(C) negatively skewed; IQR is approximately $700
(D) positively skewed; IQR is approximately $2000
(E) negatively skewed; IQR is approximately $2000

Answer

2. A researcher is studying an old edition of an encyclopedia. She wishes to estimate the proportion of the printed matter in the encyclopedia that is diagrams and pictures (as opposed to text). The encyclopedia consists of 30 volumes, and she observes that the proportion of printed matter that is diagrams and pictures is roughly the same in each of the volumes. The researcher randomly selects four of the volumes, and then studies every page in those four volumes. This is an example of which type of sampling?

(A) Cluster
(B) Convenience
(C) Simple random
(D) Stratified random
(E) Systematic

Answer

3. A company produces cloth for use in airplane seats. The company claims that the mean breaking strength μ for specimens of the cloth is 80 pounds of force, but the airlines who buy the cloth are concerned that the cloth might be weaker than that. A group working on behalf of the airlines takes a random sample of specimens of the cloth and finds the breaking strength of each specimen in the sample. What hypotheses should the group use to test the manufacturer's claim?

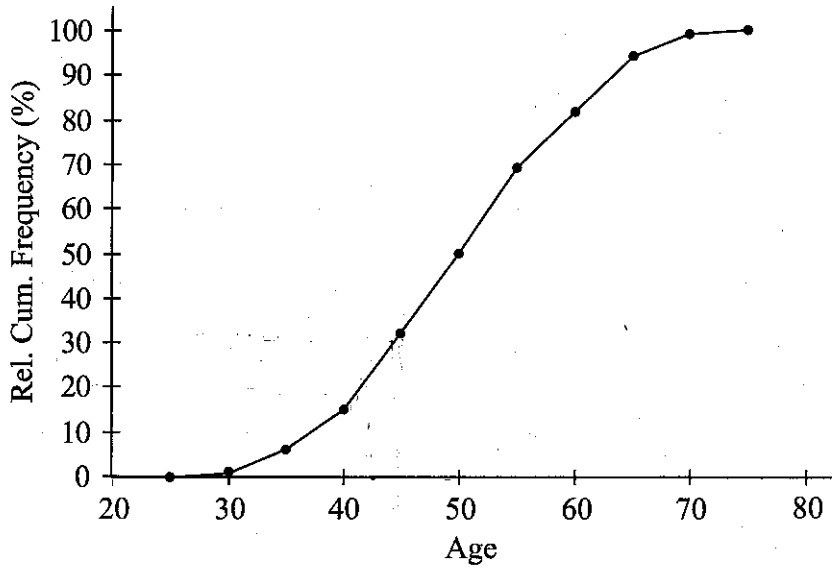
(A) $H_0: \mu = 80, \quad H_a: \mu < 80$
(B) $H_0: \mu = 80, \quad H_a: \mu \neq 80$
(C) $H_0: \mu = 80, \quad H_a: \mu > 80$
(D) $H_0: \mu < 80, \quad H_a: \mu = 80$
(E) $H_0: \mu > 80, \quad H_a: \mu = 80$

Answer

4. In a company, 78% of the employees opt for medical insurance and 42% of the employees opt for life insurance. 82% of the employees opt for at least one of these benefits. What percent of the employees opt for both of these benefits?

(A) 4%
(B) 18%
(C) 33%
(D) 38%
(E) 40%

Answer



5. A society has 160 members. A relative cumulative frequency graph of their ages is shown in the figure above. Approximately how many of the society's members are over 43 years old?

(A) 25
(B) 40
(C) 48
(D) 75
(E) 120

Answer

6. A track and field coach wants to find out whether a particular hammer thrower performs better, on average, in the morning or in the afternoon. The coach observes a random sample of the athlete's morning throws and a random sample of the athlete's afternoon throws. Which one of the following significance tests could be used to analyze the results?

(A) One sample t-test for a mean
(B) Two sample t-test for means
(C) Paired t-test
(D) One proportion z-test
(E) Two proportion z-test

Answer

7. Diana has several children and each of her children has several friends, so she can never be sure how many children will come to dinner. However, over long experience she has worked out that the probability distribution for the number of children who will come to dinner is as shown below.

Number of children	0	1	2	3	4	5	6	7	8	9	10
Probability	0.01	0.04	0.13	0.15	0.16	0.17	0.12	0.09	0.07	0.04	0.02

On any given evening, what is the minimum number of places that she should set at the dinner table for the children in order to be at least 80% sure that all the children can be seated?

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

Answer

8. A machine produces metal springs for computer lids. Over a long period of time it has been found that 10% of the springs produced by the machine are defective. After some adjustments to the machine, a random sample of 200 springs is selected and it is found that 16 of the springs in the sample are defective. The appropriate significance test is carried out in order to determine whether the proportion of defective springs has changed. Which of the following is the correct p-value for the test?

(A) $2 \cdot P\left(z < \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(0.1)(0.9)}{200}}}\right)$

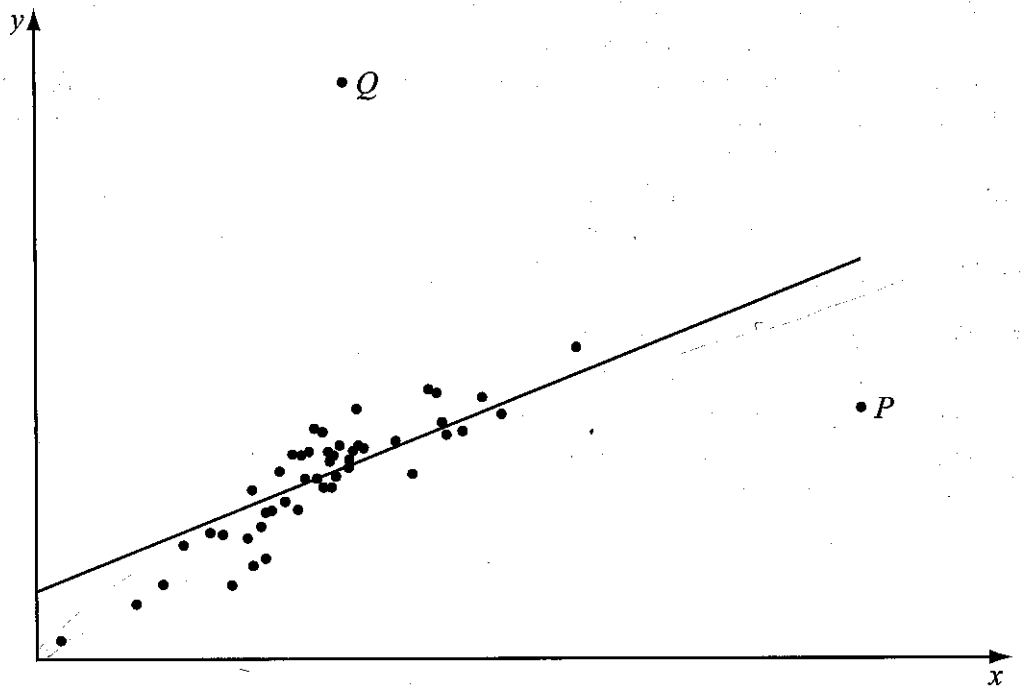
(B) $2 \cdot P\left(z > \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(0.1)(0.9)}{200}}}\right)$

(C) $2 \cdot P\left(z < \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(.08)(.92)}{200}}}\right)$

(D) $2 \cdot P\left(z > \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(.08)(.92)}{200}}}\right)$

(E) $\dfrac{1}{2} \cdot P\left(z < \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(.08)(.92)}{200}}}\right)$

Answer



9. The scatterplot above shows 52 points with the associated least squares regression line for predicting values of y from values of x. One of the two labeled points – either P or Q – will be removed. Which of the following is true?

(A) Removal of the point P would substantially increase the slope of the least squares regression line. Removal of the point Q would have little effect on the slope of the least squares regression line.

(B) Removal of the point P would substantially decrease the slope of the least squares regression line. Removal of the point Q would have little effect on the slope of the least squares regression line.

(C) Removal of the point Q would substantially increase the slope of the least squares regression line. Removal of the point P would have little effect on the slope of the least squares regression line.

(D) Removal of the point Q would substantially decrease the slope of the least squares regression line. Removal of the point P would have little effect on the slope of the least squares regression line.

(E) Removal of the point P would have a substantial effect on the slope of the least squares regression line and removal of the point Q would have a substantial effect on the slope of the least squares regression line.

Answer

10. In a particular country it is known that 40% of the residents have blue eyes, 35% of the residents have brown eyes, and 25% of the residents have green eyes. A student carries out a study to determine whether, in terms of color, the eyes of dolls manufactured in that country are representative of the residents of the country. The student takes a random sample of 40 brands of doll, and finds that 10 of them have blue eyes, 19 of them have brown eyes, and 11 of them have green eyes. She then carries out the appropriate significance test and obtains the p-value for the test. Which of the following is true?

(A) The p-value is between 0 and 0.05.
(B) The p-value is between 0.05 and 0.1.
(C) The p-value is between 0.1 and 0.15.
(D) The p-value is between 0.15 and 0.2.
(E) The p-value is greater than 0.2.

Answer

11. A machine at a casino returns money to the player on 20% of plays. A player makes plays on the machine repeatedly, hoping to get a return. Assuming that the outcomes of the plays are independent, what is the probability that it takes at least three plays for the player to get a return?

(A) 0.128
(B) 0.312
(C) 0.488
(D) 0.512
(E) 0.640

Answer

12. A vending machine delivers varying amounts of coffee. The standard deviation of the amount per serving is known, but the mean amount per serving has recently been adjusted to an unknown value. The person responsible for the machine takes ten servings of coffee from the machine, and is willing to assume that these ten servings form a random sample. She calculates the sample mean serving size to be 10.8 ounces.

    She now intends to carry out a z-test about the mean serving size μ for all servings of coffee from this machine. Which of the following pairs of hypotheses will result in the smallest p-value?

    (A) $H_0: \mu = 10, \quad H_a: \mu < 10$
    (B) $H_0: \mu = 10, \quad H_a: \mu > 10$
    (C) $H_0: \mu = 11, \quad H_a: \mu < 11$
    (D) $H_0: \mu = 11, \quad H_a: \mu \neq 11$
    (E) $H_0: \mu = 11, \quad H_a: \mu > 11$

Answer

13. A student named Russell does a survey concerning the amount of sleep his fellow students are getting. Having taken a sample of students and asked each student the total amount of sleep he/she has had over the past week, he discovers that one of the responses is an outlier. Russell strongly suspects that this response was untrue, but he does not feel able to exclude it from his data set. In order to summarize the center and the spread of the complete set of responses he should quote the

    (A) mean and the standard deviation
    (B) mean and the interquartile range
    (C) mean and the range
    (D) median and the interquartile range
    (E) median and the range

Answer

14. In a high school there are 638 underclassmen (9th and 10th graders) and 523 upperclassmen (11th and 12th graders). Of the underclassmen 83.1% take the bus to school, and of the upperclassmen 70.9% take the bus to school. If a student is chosen at random from those students who take the bus to school, what is the probability that this student is an underclassman?

(A) 0.457
(B) 0.550
(C) 0.588
(D) 0.671
(E) 0.831

Answer

15. A survey is conducted to compare the proportions of men and women who access their bank statements online. Denoting the population proportions by p_M and p_W, a two-proportion z-test is carried out to test $H_0: p_M = p_W$ against $H_a: p_M > p_W$. The value of the test statistic is found to be $z = 0.784$, and the p-value for the test is found to be 0.216. Which of the following is a correct interpretation of the p-value?

(A) Given the results of the survey, the probability that $p_M > p_w$ is 0.216.
(B) Given the results of the survey, the probability that $p_M = p_W$ is 0.216.
(C) Given that $p_M = p_W$, the probability of getting a value of z at least as large as 0.784 is 0.216.
(D) Given that $p_M > p_W$, the probability of getting a value of z at least as large as 0.784 is 0.216.
(E) Given that $p_M \neq p_W$, the probability of getting a value of z at least as large as 0.784 is 0.216.

Answer

16. A school district currently allows 12th graders at the high school to drive to school. The Board of Education is considering withdrawing this policy, and wishes to determine the opinions of the parents of students in grades K-12 on the issue.

    The Board has a list of email addresses covering the parents of most of the students in the district. An email containing the following message is sent to the parents on the list.

> Please read the following statement:
>
> "12th graders should not be allowed to drive to school. The reduction in parking would allow for a substantial expansion in student activities."
>
> Do you strongly agree, agree, disagree, strongly disagree, or have no opinion? Please reply with your response.

    After three days, the responses are gathered and are analyzed. Which of the following could NOT be considered a source of bias in this study?

    (A) The statement is worded in a way that is likely to influence the reader in a particular direction.
    (B) The message is sent to parents of students who are not in the 12th grade.
    (C) The list of email addresses does not include the parents of all students in the district.
    (D) Some parents will not read the email within the three-day period.
    (E) Some parents who read the email will choose not to respond.

Answer

17. For a group of students, the correlation between their heights (in inches) and their weights (in pounds) is 0.332. You are given that 1 inch = 2.54 centimeters and that 1 pound = 0.454 kilogram. If the heights are expressed in centimeters and the weights are expressed in kilograms, what will be the value of the correlation?

    (A) 0.059
    (B) 0.288
    (C) 0.332
    (D) 0.383
    (E) 1.857

Answer

18. A track and field coach has observed two javelin throwers for a long period of time, and now has to select one of them for the team. Which of the following would NOT be a good reason to choose thrower A in preference to B?

(A) The mean for thrower A is greater than the mean for thrower B.
(B) The median for thrower A is greater than the median for thrower B.
(C) The third quartile for thrower A is greater than the third quartile for thrower B.
(D) The maximum for thrower A is greater than the maximum for thrower B.
(E) The distribution of A's throws is positively skewed whereas the distribution of B's throws is roughly symmetrical.

Answer

19. A company is developing a new drug for reducing the symptoms of pollen allergies. They have developed two forms of the drug: A and B. The company wants to find out which form of the drug is most effective and to determine whether the amount to be taken each day should be split into one, two, or three doses. A set of volunteers who suffer from pollen allergies is split into six groups to receive treatments according to the following table.

	1 Dose	2 Doses	3 Doses
Drug A			
Drug B			

How many explanatory variables (factors) are there in this experiment?

(A) 1
(B) 2
(C) 3
(D) 5
(E) 6

Answer

20. An airline observes a random sample of its flights on a particular route. The 95% confidence interval for the mean time (in minutes) for all flights on this route is calculated to be (47.0, 53.0). Which of the following is NOT true?

(A) At the 95% confidence level, the true mean flight time is within 3.0 minutes of the sample mean flight time.
(B) If the true mean flight time were outside the interval (47.0, 53.0) then the sample mean that was found would be very unlikely.
(C) Approximately $2\frac{1}{2}\%$ of flights on this route are longer than 53 minutes.
(D) We are 95% confident that the true mean flight time is between 47.0 and 53.0 minutes.
(E) If many random samples of the same size were taken and the 95% confidence intervals were calculated, then 95% of the confidence intervals would contain the true mean flight time.

Answer

21. A "population" is formed by placing five balls in a bag. The balls are labeled 1, 2, 3, 4, and 5, respectively. The mean of this population is $\mu = 3$. Someone who does not know the contents of the bag will estimate the value of μ by randomly taking a sample of three of the balls (without replacement) and finding either the sample mean or the sample median.

    In the meantime, a statistician has listed all the possible samples of size three (sampling without replacement) and has calculated the sample mean and the sample median for each possible sample. The statistician finds that:

    - All the possible sample means form a distribution whose mean is 3 and whose standard deviation is 0.577.
    - All the possible sample medians form a distribution whose mean is 3 and whose standard deviation is 0.775.

    Regarding the choice between using the sample mean and using the sample median for estimating μ, which of the following is true?

    (A) Both the sample mean and the sample median are unbiased, but the sample median is preferable as it has the larger standard deviation.
    (B) Both the sample mean and the sample median are unbiased, but the sample mean is preferable as it has the smaller standard deviation.
    (C) The sample mean is unbiased and the sample median is biased, so the sample mean is preferable.
    (D) The sample median is unbiased and the sample mean is biased, so the sample median is preferable.
    (E) Both the sample mean and the sample median are biased.

Answer

22. A regular deck of cards has four suits each consisting of thirteen cards. One of the suits is "hearts" and one card in each suit is a "queen." One card is going to be picked at random from the deck. Let A be the event that the card is a heart and let B be the event that the card is a queen. Which of the following is true?

(A) The events A and B are independent and mutually exclusive.
(B) The events A and B are independent but not mutually exclusive.
(C) The events A and B are not independent but are mutually exclusive.
(D) The events A and B are not independent and not mutually exclusive.
(E) It is not possible to tell from the information given whether or not the events A and B are mutually exclusive.

Answer

23. A new warm-up procedure has been suggested for use before working out, and it is hoped that the procedure will encourage a greater increase in muscle mass. In order to test this, a study is designed using 40 volunteers who already work out regularly.

The volunteers will be randomly split into two groups, each of size 20. The first group will be taught the warm-up exercises and will be supervised doing the exercises prior to their regular workouts. The second group will merely continue with their regular workouts. At the beginning and at the end of the study, the muscle mass of each of the volunteers will be measured by people who do not know which volunteers were in which group.

Which of the following is NOT the case in the study described?

(A) This study is an experiment.
(B) Randomization is used.
(C) A control group is used.
(D) The study is conducted in a double-blind manner.
(E) There is no blocking involved in the study.

Answer

24. A set of scores has mean 70.3 and a standard deviation 8.8. The scores are now scaled according to the formula $y = 0.7x + 30$, where x is the old score and y is the new score. What is the standard deviation of the new scores?

(A) 4.31
(B) 6.16
(C) 7.36
(D) 36.16
(E) 37.36

Answer

25. In a high school, all of the 11th graders take both math and physics. After the students have taken the midyear exam in both subjects, the physics teachers are considering the results, and have found the value of r^2, the square of the correlation coefficient between the math scores and the physics scores. Which of the following is best answered by consideration of the value of r^2?

(A) Whether high physics scores are associated with high math scores
(B) Whether the relationship between physics scores and math scores would be better represented by a curve or a straight line
(C) To what extent the variation in physics scores can be explained by a linear relationship between physics scores and math scores
(D) Whether there is an outlier in the scatterplot of physics scores and math scores
(E) Whether the physics scores are on the whole higher than the math scores

Answer

26. In a random sample of 400 adults, each person stated his or her political preference. The sex (male/female) of each respondent was also noted. The results are shown in the table below.

	Democrat	Republican	Other
Male	94	78	18
Female	88	86	36

If political preference is independent of sex, which of the following is the expected number of respondents who are female and support the Democratic Party?

(A) 40.04
(B) 46.20
(C) 86.45
(D) 95.55
(E) 103.895

Answer

27. Suppose that an observational study has shown that people who regularly consume substantial amounts of olive oil live longer lives, on average, than those who do not. Of the following arguments, which is strongest in explaining why the result of the study does <u>not</u> imply that in order to live longer one should start to regularly consume substantial amounts of olive oil?

(A) Olive oil is high in fat, and it's not a good idea to eat high-fat foods.
(B) There are many other factors contributing to how long you live that were not considered by the study.
(C) If a person is recorded as eating substantial amounts of olive oil and living a long life, we don't know whether the long life was caused by the olive oil eating or, for example, regular exercise.
(D) People who choose to include substantial amounts of olive oil in their diets might well be the sort of people who have healthier lifestyles in general, and a healthy lifestyle leads to a long life.
(E) Olive oil is associated with frying, and frying is unhealthy.

Answer

28. A team of psychologists is studying the behavior of the students in a first grade class. There are 16 girls and 16 boys in the class, and for each student the psychologists record the number of minutes "on task" during a forty minute class. The team wishes to compare the on-task times of the girls with the on-task times of the boys. Which of the following would NOT be a suitable graph for displaying the results?

(A) Parallel dotplots with equal scales
(B) Back-to-back stemplot
(C) Histograms with equal scales
(D) Side-by-side boxplots
(E) Scatterplot with girls' times plotted as x-values and boys' times plotted as y-values

Answer

29. The amount of flour per bag delivered by a machine is known to have a standard deviation of 0.4 ounce. What is the minimum sample size required to estimate the mean amount of flour per bag to within 0.1 ounce with 95% confidence?

(A) 3
(B) 8
(C) 43
(D) 62
(E) 154

Answer

30. In the context of z- and t-tests for the mean using small samples, which of the following is (are) true?

I. The z-test requires the assumption that the population distribution is normal.
II. The t-test requires the assumption that the population distribution is normal.
III. The t-test is used when the population standard deviation is unknown.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Answer

31. When a large number of a particular type of seed is planted, it is known that 70% of the seeds will germinate. In addition, the germination of any one seed is independent of the germination of any other seed. If 20 of the seeds are planted, what are the mean and the standard deviation of the number of seeds that germinate?

(A) mean = 0.7, standard deviation = 0.102
(B) mean = 0.7, standard deviation = 0.458
(C) mean = 14, standard deviation = 0.102
(D) mean = 14, standard deviation = 2.049
(E) mean = 14, standard deviation = 4.2

Answer

32. A company has a machine that produces cans of coconut milk, and it has been noticed that the amount of coconut milk varies from can to can. The amounts are normally distributed with standard deviation 8 milliliters. The label used on the cans states that each can contains 414 milliliters. The management of the company decides to set the mean μ of the amount of coconut milk per can so that 95% of the cans contain more than 414 milliliters. Of the following, which is the closest approximation to μ in milliliters?

(A) 400
(B) 415
(C) 420
(D) 425
(E) 430

Answer

33. A political party wishes to estimate the proportion of voters that support the party in a particular state. The party will poll a random sample of n voters from the state. Which of the following is likely to result in the smallest margin of error?

(A) $n = 400$, confidence level = 95%
(B) $n = 400$, confidence level = 98%
(C) $n = 400$, confidence level = 99%
(D) $n = 500$, confidence level = 95%
(E) $n = 500$, confidence level = 99%

Answer

34. A pharmaceutical company wishes to compare the effectiveness of three drugs, A, B, and C, that are designed to reduce blood pressure. The company believes that the younger a person is, the more likely he is to respond to a drug of this sort. The company intends to design an experiment in which each subject will be instructed to take one of the drugs regularly for a four week period. Each subject's blood pressure will be measured at the beginning and at the end of the four week period.

There are three young men, three middle-aged men, and three elderly men available to take part in this study. Which of the following is the most appropriate method for assigning the treatment groups?

(A) For each man, randomly choose which drug he will be given.
(B) From the whole set of nine men, randomly choose three to receive drug A, three to receive B, and three to receive C.
(C) For the three young men, randomly assign one man to drug A, one man to drug B, and one man to drug C; repeat this process for the middle-aged men and for the elderly men.
(D) Randomly choose one of the drugs, and give that drug to all the young men; randomly choose one of the remaining drugs and give that to all the middle-aged men; and then give the third drug to all the elderly men.
(E) Randomly pick one man from each age group and from these three randomly assign one to drug A, one to drug B, and one to drug C; then pick another man from each age group and do the same thing; then do the same thing for the remaining three men.

Answer

35. A manufacturer of tires has used a particular type of rubber for a long time, and has established over the years that the mean life of the tires is 40,000 miles. However, the company has now changed the type of rubber used and needs to find out whether the mean life of the tires has changed. Having tested a random sample of the tires, a *t*-test for the mean is carried out using $H_0: \mu = 40,000$ *versus* $H_a: \mu \neq 40,000$. The *t*-value for the test is found to be -1.902 and the *p*-value is found to be 0.063. Using a 5% significance level, which of the following is a correct conclusion for the test?

(A) Since $p > 0.05$ we do not have sufficient evidence to conclude that the mean life of the tires is not equal to 40,000 miles.
(B) Since $p > 0.05$ we do not have sufficient evidence to conclude that the mean life of the tires is less than 40,000 miles.
(C) Since $p > 0.05$ we have sufficient evidence to conclude that the mean life of the tires is equal to 40,000 miles.
(D) Since $p > 0.05$ we have sufficient evidence to conclude that the mean life of the tires is not equal to 40,000 miles.
(E) Since $p > 0.05$ we have sufficient evidence to conclude that the mean life of the tires is greater than 40,000 miles.

Answer

36. A very large population has standard deviation denoted by σ. A random sample of size n will be taken from this population. The quantity $\frac{\sigma}{\sqrt{n}}$ is

(A) the mean of the distribution of the sample standard deviation
(B) the standard deviation of the distribution of the sample mean
(C) the standard deviation of the sample
(D) an estimate of the population standard deviation
(E) an estimate of the sample standard deviation calculated from the population standard deviation

Answer

37. In a test of the null hypothesis $H_0: \mu = 50$ against the alternative hypothesis $H_a: \mu < 50$, with significance level α using sample size n, which of the following is the smallest?

(A) The probability of Type II error when $\mu = 48$, given that $n = 40$ and $\alpha = 0.05$
(B) The probability of Type II error when $\mu = 46$, given that $n = 40$ and $\alpha = 0.05$
(C) The probability of Type II error when $\mu = 48$, given that $n = 40$ and $\alpha = 0.01$
(D) The probability of Type II error when $\mu = 46$, given that $n = 40$ and $\alpha = 0.01$
(E) The probability of Type II error when $\mu = 48$, given that $n = 20$ and $\alpha = 0.05$

Answer

38. Having graded a test, a teacher was interested in the relationship between the amount of time the students studied for the test and the scores they received. She asked the 24 students individually how much they studied, and then compiled a list giving for each student the amount of time studied and the score on the test. The teacher performed a least squares regression analysis. Part of the computer output from that analysis is shown below.

Dependent variable: Score on test

Predictor	Coef	SE Coef	T	P
Constant	69.555194	3.721432	18.69	<.0001
Time	0.2642443	0.109216	2.42	0.0243

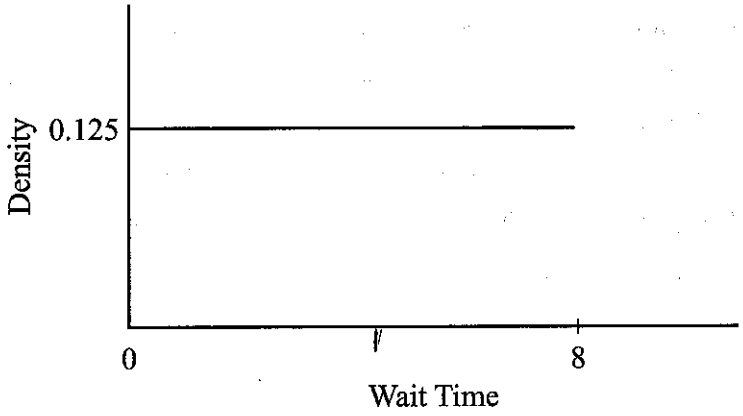
S = 6.3241 R-sq = 21.0% R-sq (adj) = 17.5%

Which of the following is a 99% confidence interval for the slope of the regression line that relates the time spent studying and the score on the test?

(A) $69.555 \pm (2.807)(3.721)$
(B) $69.555 \pm (2.819)(3.721)$
(C) $69.555 \pm (18.69)(3.721)$
(D) $0.264 \pm (2.807)(0.109)$
(E) $0.264 \pm (2.819)(0.109)$

Answer

39. Every afternoon, Jennifer waits for a subway train. The density curve for the amount of time she has to wait (in minutes) is shown in the diagram below.

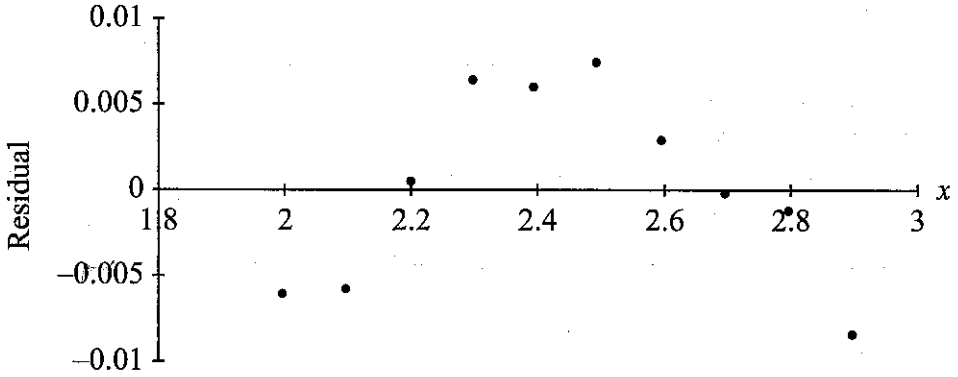


The mean and standard deviation of the wait time are 4 and 2.309, respectively. If a random sample of 40 afternoons is taken, what is the approximate probability that Jennifer's sample mean wait time is less than 5 minutes?

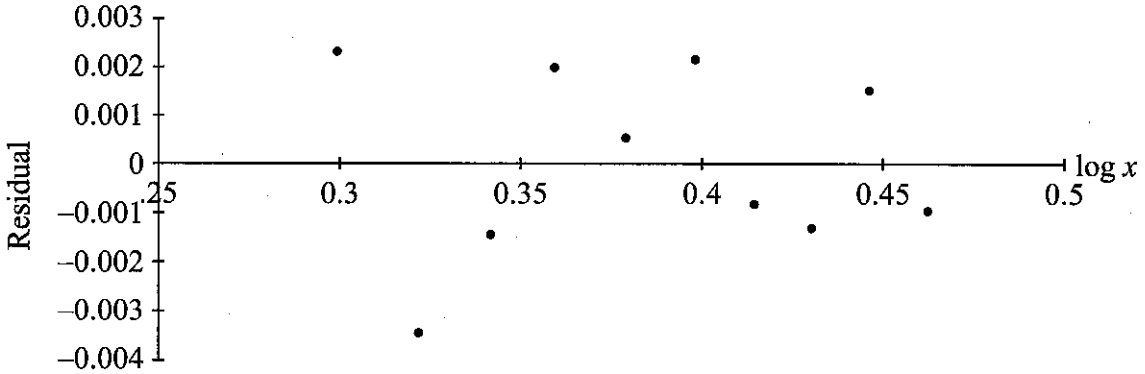
(A) 0.003
(B) 0.332
(C) 0.625
(D) 0.668
(E) 0.997

Answer

40. Two variables, x and y, were measured for a random sample of 10 subjects. In the first of two transformations, $\log y$ was plotted (on the vertical axis) against x (on the horizontal axis), a least squares regression was performed on the transformed variables, and the following residual plot was obtained.



In the second transformation, $\log y$ was plotted (on the vertical axis) against $\log x$ (on the horizontal axis), a least squares regression was performed on the transformed variables, and the following residual plot was obtained.



Which of the following conclusions is best supported by the evidence above?

(A) x and y are related according to an equation of the form $y = ax^p$, where a and p are constants.

(B) x and y are related according to an equation of the form $y = a + x^p$, where a and p are constants.

(C) x and y are related according to an equation of the form $y = a \cdot b^x$, where a and b are constants.

(D) x and y are related according to an equation of the form $y = a + b^x$, where a and b are constants.

(E) x and y are related according to an equation of the form $y = a + b\log x$, where a and b are constants.

Answer

Sample Examination Two

SECTION I
Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A school administrator in a large high school thinks that an SAT preparation course will improve the mean performance of his students on the SAT exam. Five hundred of his seniors who have not yet taken the exam volunteer to take part in a study to test the administrator's theory. He randomly selects half of the 500 students to take the prep course. When the course is over, all 500 students take the SAT. The scores of those who took the prep course are compared to the scores of those who did not take the prep course.

   This study would be classified as

   (A) a survey for which no causal inference could be drawn about the effectiveness of an SAT preparation course for the seniors at this school
   (B) an observational study for which the results are valid in determining whether an SAT preparation course is effective in improving the scores of the seniors at this school
   (C) an observational study for which the results are only valid in determining whether an SAT preparation course is effective in improving the scores of the 500 seniors in this study
   (D) an experiment for which results are only valid in determining whether an SAT preparation course is effective in improving the scores of the 500 seniors in this study
   (E) an experiment for which results are valid in determining whether an SAT preparation course is effective in improving the scores of all seniors in this school

Answer

(c) Use the random number table given below and your assignment of digits from part (b) to simulate the responses from one sample of 20 students. Show your work clearly on the table, and note the total score for your sample.

4	4	7	3	8	8	5	3	5	1	7	7	6	7	8
6	3	7	6	4	2	5	9	3	9	5	3	8	8	9
6	6	4	8	5	2	5	2	9	9	4	6	9	8	8
1	6	9	0	9	9	0	1	0	4	9	7	4	8	0

The assistant runs the simulation in part (c) 200 times, and obtains the following results.

Total Score	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Number of Runs	1	3	4	3	8	9	16	13	26	11	17	15	28	16	15	6	5	2	0	1	1

(d) On the basis of this set of results (which is based on the assumption that the three responses are favored by equal proportions of the school), do you think that a total score of 24 in a sample of 20 students gives convincing evidence that the student body as a whole is in favor of the idea? Explain carefully the logic behind your answer.

2. Using a hypothesis test, an airline wishes to provide convincing evidence that its airplanes are, on average, less than 8 minutes late in landing. Denoting the mean amount of time by which its airplanes are late in landing by μ, the hypotheses that should be used are

(A) $H_0: \mu < 8, \quad H_a: \mu = 8$
(B) $H_0: \mu = 8, \quad H_a: \mu \neq 8$
(C) $H_0: \mu = 8, \quad H_a: \mu < 8$
(D) $H_0: \mu = 8, \quad H_a: \mu > 8$
(E) $H_0: \mu < 8, \quad H_a: \mu \geq 8$

Answer

3. A student takes three standardized tests. She scores 600 on each of the tests. The scores of all students who took the tests are summarized in the table below.

	Mean	**Standard Deviation**
Test I	500	80
Test II	470	120
Test III	560	30

Using standardized scores (z-scores), which of the following gives the ranking of her performances on the three tests, from best to worst?

(A) I, II, III
(B) III, II, I
(C) I, III, II
(D) III, I, II
(E) II, I, III

Answer

4. A test has 10 multiple-choice questions each with four possible answers. You are to design a simulation to estimate the probability of a student answering at least three questions correctly if the student guesses on every question. Which of the following would be an appropriate assignment of the digits in a random number table to accomplish this task?

   (A) Choose the numbers 0, 1, 2, 3, 4 to represent a correct answer, and 6, 7, 8, 9 to represent an incorrect answer.
   (B) Choose the numbers 0, 1, 2 to represent a correct answer and 3, 4, 5, 6, 7, 8 or 9 to represent an incorrect answer.
   (C) Choose the numbers 0, 1, 2, 3 to represent a correct answer and the numbers 4, 5, 6, 7, 8, 9 to represent an incorrect answer.
   (D) Choose the number 0 to represent a correct answer and the numbers 1, 2, 3 to represent an incorrect answer. Ignore all other digits.
   (E) Choose the numbers 00 – 25 to represent a correct answer and the numbers 26 – 99 to represent an incorrect answer.

Answer

5. A die with six faces has three 1's, two 2's and one 3. If a pair of these die are rolled together, what is the expected number of 2's in one roll of the pair of die?

   (A) 1/9
   (B) 2/9
   (C) 1/3
   (D) 5/9
   (E) 2/3

Answer

6. The correlation between the variables x and y is 0.3. Which of the following would change the value of the correlation to -0.3?

   (A) Subtracting a value larger than $\bar{x}$ from each of the x-values
   (B) Switching the values of x with the values of y
   (C) Subtracting $\bar{x}$ from the each of the values of x and subtracting $\bar{y}$ from each of the values of the y
   (D) Multiplying the x-values by -1 and the y-values by -1
   (E) Multiplying the x-values by -1

Answer

7. What is the effect of multiplying every element in a data set by an integer k?

   (A) There is no change in the mean and there is no change in the variance.
   (B) The variance is divided by $\sqrt{k}$ and the mean is multiplied by k.
   (C) The mean is multiplied by k and the variance is multiplied by k^2.
   (D) The mean is multiplied by k and the variance is multiplied by $\sqrt{k}$.
   (E) One cannot determine the effect on the mean and the variance of the data set.

Answer

8. A researcher wants to determine if there is a difference in the effectiveness of two vaccines, A and B. Subjects were assigned at random to the two vaccines. All of the subjects were then exposed to a virus. Of the 210 people who were assigned to vaccine A, 84 were infected, and 77 of the 220 assigned to vaccine B were infected. Which of the following would be used to compute a 95% confidence interval for the difference between the proportion of people infected when injected with vaccine A and the proportion of people infected when injected with vaccine B?

(A) $.05 \pm 1.96 \times \sqrt{0.37 \times 0.63\left(\frac{1}{210} + \frac{1}{220}\right)}$

(B) $.05 \pm 1.96 \times \sqrt{0.4 \times 0.6\left(\frac{1}{210} + \frac{1}{220}\right)}$

(C) $.05 \pm 1.96 \times \sqrt{\frac{0.4 \times 0.6}{210} + \frac{0.35 \times 0.65}{220}}$

(D) $.05 \pm 1.96 \times \sqrt{\left(\frac{0.4}{210} + \frac{0.35}{220}\right)}$

(E) $.05 \pm 1.96 \times \frac{0.4 - 0.35}{\sqrt{430 \times 0.37 \times 0.67}}$

Answer

9. A large college statistics class has 400 students. The students' final exam scores in the class are normally distributed with a mean of 80 and a standard deviation of 6. What is the highest score that the top 60 students in the class exceeded?

(A) 81
(B) 83
(C) 86
(D) 90
(E) 95

Answer

10. In conducting a Chi-Square test of association between gender and grade, what is the expected count for the number of males who earned a grade of B?

	Grade			
	A	B	C	D
Male	10	32	25	2
Female	5	41	14	12

(A) $\frac{73 \cdot 69}{141}$

(B) $\frac{73 \cdot 69}{110}$

(C) $\frac{32 \cdot 73}{141}$

(D) $\frac{32 \cdot 69}{110}$

(E) $\frac{69 \cdot 32}{141}$

Answer

11. A student wishes to know if SUV drivers at his school are more likely to be male than female. He takes a simple random sample of 60 students from a large list of students that consists of all the SUV drivers at the school, and records the gender of each student in the sample. What would be the appropriate inference procedure to use?

(A) Two sample *t*-test; testing to see if the mean number of females differs from the mean number of males driving SUV's
(B) Chi-square test of independence
(C) Test for a population proportion; testing to see if the proportion of SUV drivers who are males is greater than 50%
(D) Test for the difference of two proportions; testing to see if the proportion of SUV drivers who are male is greater than the proportion of SUV drivers who are female
(E) Paired *t*-test

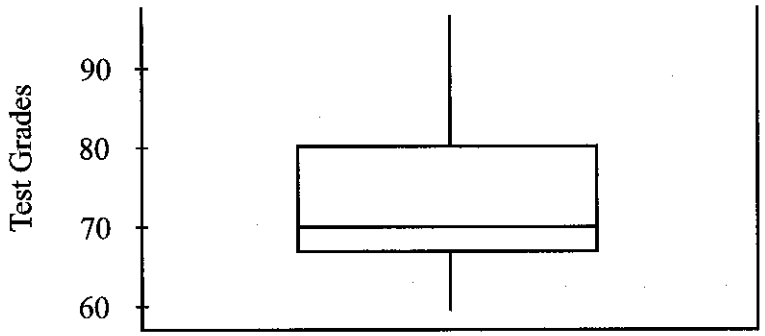
Answer

12. A random sample is taken from a population and a 90% confidence interval for population mean is calculated. If the confidence level is increased to 95%, the confidence interval for μ

(A) becomes narrower
(B) becomes 0.05
(C) does not change
(D) becomes wider
(E) becomes 0.025

Answer

13. A small school has 40 algebra students. A boxplot shown below was constructed from the final exam scores of all forty students.



Which of the following statements is/are true about the distribution of exam grades?

I. The distribution of the 40 grades is skewed to the left.
II. The mean of the distribution is larger than 70.
III. The interquartile range is less than 10.
IV. Fifty percent of the students received scores greater than 70 on the final exam.

(A) II only
(B) IV only
(C) II and IV only
(D) I, II, and IV
(E) II, III, and IV

Answer

14. A student is interested in the effects of different walking styles on heart rate. He decides to use 30 volunteers from his school for his experiment. All 30 participants find their at-rest pulse rates. Each participant walks twice, for 10 minutes each: once using a fast pace with no arm movement and once using a fast pace with an exaggerated arm movement. The student throws a coin to determine which style of walking each participant will use first. All participants get sufficient rest between the walks to let their pulse rates return to normal. The student then compares the mean increase (calculated over all the participants) in pulse rate as a result of the walk with no arm movement to the mean increase (calculated over all the participants) in pulse rate as a result of the walk with exaggerated arm movement.

    Which of the following statements is/are true?

    I. This is an experiment.
    II. This study uses a matched-pairs design.
    III. A two-sample *t*-test could be used to analyze the results of the study.

    (A) III only
    (B) I and II only
    (C) II and III only
    (D) I, II and III
    (E) None of the statements is true

    Answer

15. A statistics student wishes to compare the strengths of various brands of paper towel. He chooses 5 brands and selects 6 towels from each brand. He numbers the towels 1-30. He then randomly selects a towel and places it in a device to hold the towel taut. Exactly 10 ml of water and a large weight are placed in the center of the towel and the time it takes for the towel to break is recorded. In this case the explanatory variable is the

    (A) amount of time it takes for the towel to break
    (B) 10 ml of water and the large weight
    (C) brand of paper towels
    (D) large weight
    (E) number of paper towels used in the experiment

    Answer

16. For the students in a statistics course, a linear regression equation was computed to predict the final exam score based on the score on the first test of the term. The equation was: $\hat{y} = 25 + 0.7x$ where y is the final exam score and x is the score on the first test. George scored 80 on the first test. On the final exam George scored 85. What is the value of George's residual?

(A) -4
(B) 4
(C) 5
(D) 81
(E) None of the above

Answer

17. Which of the following statements about the sampling distribution of a sample mean is/are true?

I. The larger the sample size, the smaller the spread of the sampling distribution.
II. Provided that the population size is significantly greater than the sample size, the spread of the sampling distribution is about the same no matter what the sample size.
III. Sampling distributions from non-normal populations are approximately normal provided n is large.

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

Answer

18. In a particular country a census revealed that fifty percent of the adult population had attended college and obtained a degree. Twenty percent had attended college but had not obtained a degree. Thirty percent had not attended college. A number of years later a study was conducted to determine whether any changes in the distribution of adult education levels had occurred. Among 1000 randomly selected adults from the population it was found that 510 had attended college and obtained a degree. 160 had attended college but had not obtained a degree. The remainder had not attended college. The appropriate hypothesis test was then performed. Assuming all the conditions for inference were met, do the sample data provide convincing evidence that there had been a change in the distribution of adult education levels?

    (A) Yes, because the percentage of adults who had not attended college had decreased.
    (B) Yes, because the p-value is less than 0.01.
    (C) Yes, because the p-value is greater than 0.10.
    (D) No, because the p-value is less than 0.01.
    (E) No, because p-value is greater than 0.10.

Answer

19. The proportion of a population that supports a particular candidate will be estimated using a 97% confidence interval. What is the minimum sample size to ensure a margin of error less than 0.03?

    (A) 153
    (B) 458
    (C) 983
    (D) 1309
    (E) 5233

Answer

20. A study was conducted to investigate the relationship between the ages of cars and their mileage. Using a random sample of 30 cars, a least squares regression line was found to be an appropriate model for the relationship between age and miles driven. The results of the regression analysis are shown below:

Source	DF	Sum of Squares	Mean Square	F Ratio
Regression	1	166.24523	166.245	93.5562
Residiual	28	49.75477	1.777	
Variable	**Coefficient**	**Std Error**	**t Ratio**	**Prob**
constant	1.8636179	0.405433	4.60	<.0001
mileage	0.0000515	0.000005	9.67	<.0001
R squared = 76.965%		R squared (adjusted) = 76.14%		
s = 1.33		Sample size =30		

Which of the following should be used to compute a 95% percent confidence interval for the slope of the regression line?

(A) $1.8636179 \pm 1.96 \times 0.405433$
(B) $1.8636179 \pm 2.048 \times 1.33$
(C) $0.0000515 \pm 1.96 \times 1.33$
(D) $0.0000515 \pm 1.701 \times 0.000005$
(E) $0.0000515 \pm 2.048 \times 0.000005$

Answer

21. A study measured the, pulse rates of 92 people after they had run for 1 mile. Here are some descriptive statistics for the variable, Pulse.

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Pulse	92	80.00	76.00	78.85	17.09	1.78

Variable	Minimum	Maximum	Q1	Q3
Pulse	50.00	140.00	68.00	87.00

Which of the following is true about the distribution of the variable, Pulse?

(A) It is skewed to the left.
(B) It is symmetric.
(C) 50% of the pulse rates are greater than 50 but less than 68.
(D) Approximately 40% of the data fall within 1 standard deviation of the mean.
(E) It contains at least one outlier.

Answer

22. A candidate for mayor hires a statistician to determine the amount of support he has for the upcoming election. The statistician tests the null hypothesis that the population proportion who say they would vote for the candidate equals 0.5 against the alternative hypothesis that the population proportion is greater than 0.5. The results from a simple random sample of 91 registered voters can be found in the printout below:

```
Test of p = 0.5 vs p > 0.5
Success = for Candidate

  Variable          X      N      Sample p     Exact P-Value
For Candidate      58     91     0.637363          0.0044
```

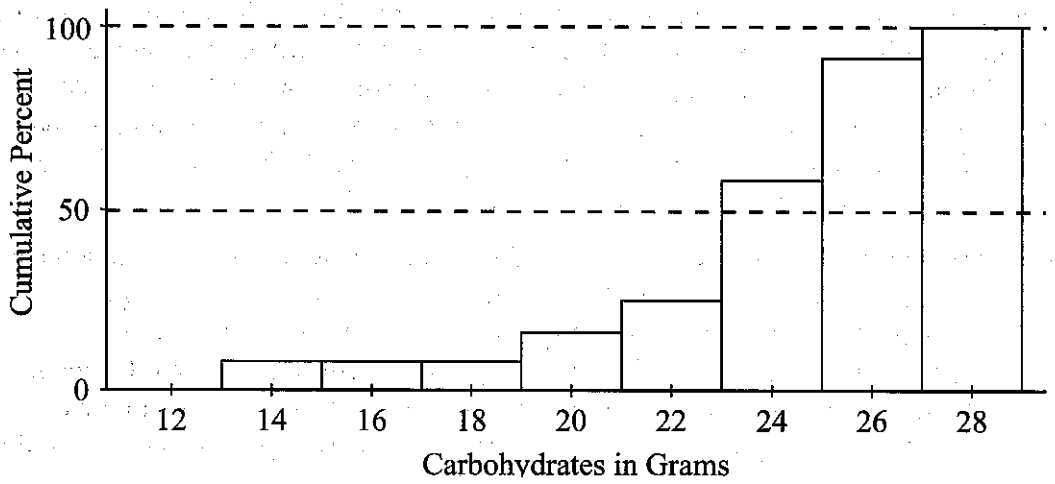
Which of the following three conclusions can be reached?

I. The candidate can be quite confident that he has support from more than 50% of registered voters.
II. The *p*-value of 0.0044 tells us that we cannot reject the null hypothesis and that the candidate has support from 50% or less of registered voters.
III. The *p*-value of 0.0044 indicates that it is not very likely to get an observed sample proportion as large as 0.637 if the null hypothesis is true.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

Answer

23. The figure below shows a cumulative relative frequency histogram of the amounts of carbohydrates (in grams per serving) in 12 brands of cereal.



Which of the following conclusions cannot be made from the graph?

(A) There is greater variability among the 6 brands of cereal with lower amounts of carbohydrates than there is among the 6 brands of cereal with higher amounts of carbohydrates.
(B) The median amount of carbohydrates for the 12 brands is at least 23 grams of carbohydrates.
(C) At least seventy-five percent of the cereals have at least 25 grams of carbohydrates.
(D) If a person is on a diet that requires less than 23 grams of carbohydrates most of these cereals would be off limits.
(E) None of these brands of cereal contains at least 15 grams and less than 19 grams of carbohydrates.

Answer

24. In a test of the hypothesis $H_0: \mu = 10$ against the alternative, $H_a: \mu \neq 10$, where μ is the population mean, the power of the test is greatest for which values of μ and n (the sample size)?

(A) $\mu = 9$ and $n = 10$
(B) $\mu = 15$ and $n = 10$
(C) $\mu = 10$ and $n = 30$
(D) $\mu = 22$ and $n = 30$
(E) $\mu = 15$ and $n = 5$

Answer

25. Suppose that you have been asked to design an experiment to test a new drug intended to reduce cholesterol against the most popular current medication. You have been told that age and gender influence the effectiveness of the drugs. Twenty men and twenty women ranging in age from 20 to 80, all with high cholesterol have volunteered to take part in the experiment. The patients will take the medication for 3 months. At the end of the 3-month period, each patient's cholesterol level will be compared to the patient's initial cholesterol level. Which of the following would be the best method to determine which drug is more effective in reducing cholesterol?

(A) Randomly assign all the subjects to take the old or the new drug, then compare the changes in cholesterol for the two groups (old drug and new drug).

(B) Block the two youngest females together, the second two youngest females together, and continue the blocking using this pattern. Do the same for the males. Within each block, randomly assign one subject to the new drug and the other to the old. Compare the changes in cholesterol for the two groups (old drug and new drug).

(C) Block the youngest male with the youngest female, the second youngest male with the second youngest female, continuing the blocking along this pattern. Within each block, randomly assign one subject to the new drug and the other to the old. Compare the changes in cholesterol for the two groups (old drug and new drug).

(D) Block the subjects by age by pairing the two youngest subjects together, the second two youngest together, and so on. Within each pair, randomly assign one subject to the new drug and the other to the old. Compare the changes in cholesterol for the two groups (old drug and new drug).

(E) Create two groups of 20 subjects each. The first group will contain all the men, the second all the women. Then randomly select which group will take the old drug and which will take the new drug. Compare the changes in cholesterol for the two groups (old drug and new drug).

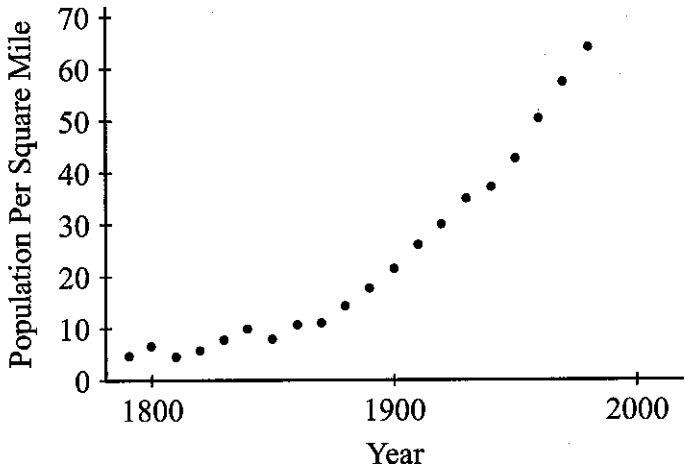
Answer

26. Suppose that the mean and standard deviation of the heights of all twelve-year-old boys in a country are 58 inches and 3.3 inches, respectively. The mean and the standard deviation of the heights of all twelve-year-old girls in the country are 56 inches and 2.6 inches, respectively. Independent random samples of 100 twelve-year-old boys and 100 twelve-year-old girls will be taken. What is the standard deviation of the sampling distribution of $\bar{x}_B - \bar{x}_G$, the difference in the sample means?

(A) 0.1765
(B) 0.42
(C) 0.7
(D) 1.33
(E) 1.765

Answer

27. For the years 1790-1980, the population per square mile in a northwest suburb can be modeled using an equation of the form $y = a \cdot b^x$, where y is the population per square mile, x is the time in years since 1790, and a and b are constants. A scatterplot showing the populations during the years 1790-1980 is shown below.



Which of the following is true?

(A) If an attempt is made at fitting a straight line to the data shown, the corresponding residual plot would show roughly a straight-line pattern.
(B) If an attempt is made at fitting a straight line to the data shown, the corresponding residual plot would show a random pattern.
(C) If an attempt is made at fitting a straight line model to the data shown, the sum of the squares of the residuals would be zero.
(D) Plotting the logarithm of the population per square mile against the year would produce an approximately linear pattern.
(E) Plotting the logarithm of the population per square mile against the logarithm of the year would produce an approximately linear pattern.

Answer

28. A large number of students at Morris Knolls High School have taken the SAT. A random sample of 25 students at the school who have taken the SAT was used to construct a 95% confidence interval for the mean SAT score of all students at the school who have taken the test. The confidence interval was found to be (900, 1100). Which of the following is true?

(A) 95% of the 25 students have mean scores between 900 and 1100.
(B) 95% of all students at Morris Knolls who have taken the test have scores between 900 and 1100.
(C) If this procedure were repeated many times, 95% of the resulting confidence intervals would contain the true mean SAT score for students at Morris Knolls who have taken the test.
(D) If this procedure were repeated many times, 95% of the sample means would be between 900 and 1100.
(E) If 100 samples of size 25 were taken from the same population and a 95% confidence interval was computed for each of the 100 samples, 5 of the intervals would be the interval from 900 to 1100.

Answer

29. For each of the years 1950-1980, the number of heart disease deaths per 100,000 people in the United States was recorded. The regression line below was computed using a statistical software package. Which statement is a correct interpretation of the slope?

```
The regression equation is
Number of deaths = 7387 - 3.63 year
```

(A) The number of heart disease deaths per 100,000 people has been dropping by about 3.63 deaths per year, on average.
(B) There is an increase of approximately 7387 deaths per year.
(C) Every 3.63 years there is a decrease on average of 1 death due to heart disease per 100,000 people.
(D) The regression line estimates that for every 3.63 years there is an average increase of 1 death due to heart disease per 100,000 people.
(E) Heart disease will be cured in the year 2036.

Answer

30. Which of the following is true?

I. If the sample size is constant, then reducing the probability of Type I error will reduce the probability of Type II error.
II. Increased power can be achieved by reducing the probability of a Type II error.
III. If the p-value of a test is 0.015, the probability that the null hypothesis is true is 0.015.

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II and III

Answer

31. The stem and leaf plot below displays the heights of 40 children in an eighth grade class.

Stem	Leaf	Count
7	0	1
6	8889	4
6	6667	4
6	444455555	9
6	22222333	8
6	0001111	7
5	899	3
5	6	1
5	5	1
5	2	1
5	1	1

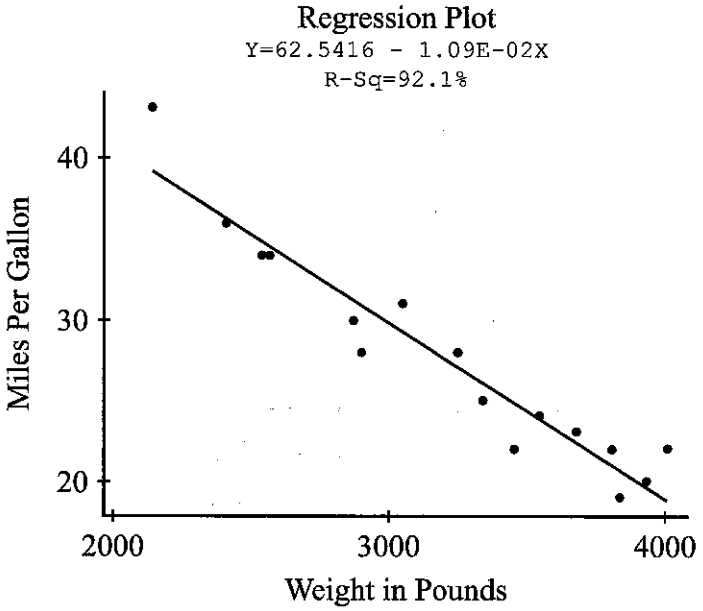
Which of the following is closest to the percentile rank of the child who is 67 inches tall?

(A) 12
(B) 15
(C) 75
(D) 85
(E) 95

Answer

32. A student was interested in the relationship between the weight of a car (in pounds) and its gas consumption (in miles per gallon). He selected sixteen different automobiles and recorded their weights along with their advertised gas consumptions. The regression equation and regression plot are shown below.

$$y = 62.5 - 0.0109x$$



What effect would the addition of the point (4,300 lbs., 15.63 mpg) have on the value of r^2?

(A) It will have no effect on r^2 because the added point lies on the least squares regression line.
(B) It will have no effect on r^2 because the added point lies at $(\bar{x}, \bar{y})$.
(C) It will decrease r^2 because the added point is an outlier.
(D) It will increase r^2 because the added point lies on the least squares line and is far from the point $(\bar{x}, \bar{y})$.
(E) It will increase r^2 because any point that is added to the scatterplot will increase the proportion of the variation in y that can be explained by the least squares line.

Answer

33. When two fair dice are rolled, what is the probability of getting a sum of 7 given that the first die rolled is an odd number?

(A) $\frac{1}{6}$

(B) $\frac{1}{9}$

(C) $\frac{1}{2}$

(D) $\frac{1}{12}$

(E) $\frac{1}{2}$

Answer

34. To determine the reliability of a specified test to determine the existence of a given disease, 300 patients were studied, 160 who had the disease and 140 who did not. The results were:

	True Status	
	Had Disease	Did Not Have Disease
Test Indicated Disease Was Present	150	15
Test Indicated Disease Was Not Present	10	125

Suppose now that the test will be used on a patient. It is not known whether or not the patient has the disease. The following hypotheses will be used:

H_0: the patient has the disease; H_a: the patient does not have the disease

According to the information given in the table, what is the probability that a Type II error will occur?

(A) 15/300
(B) 10/300
(C) 15/140
(D) 10/140
(E) 15/160

Answer

35. Suppose that 70% of the population of a city supports the policies of the current administration and 30% are against these policies. If a random sample of 1600 people from this city are interviewed, what is the probability that fewer than 448 will be against the policies of the current administration?

(A) $\binom{1600}{448}(0.7^{448})(0.3^{1152})$

(B) $1 - \binom{1600}{1152}(0.3^{448})(0.7^{1152})$

(C) $P\left(z > \dfrac{0.28 - 0.3}{\sqrt{\dfrac{0.7 \cdot 0.3}{1600}}}\right)$

(D) $P\left(z > \dfrac{0.72 - 0.7}{\sqrt{\dfrac{0.72 \cdot 0.28}{1600}}}\right)$

(E) $P\left(z > \dfrac{0.72 - 0.7}{\sqrt{\dfrac{0.7 \cdot 0.3}{1600}}}\right)$

Answer

36. The student government at a high school wants to conduct a survey of student opinion. Which of the following sampling methods will produce a simple random sample of 60 students?

(A) Survey the first 60 students to arrive at school in the morning.
(B) Survey every 10th student entering the school library until 60 students are surveyed.
(C) Use random numbers to choose 15 each of first-year, second-year, third-year, and fourth-year students.
(D) Number the cafeteria seats. Use a table of random numbers to choose seats and interview the students until 60 have been interviewed.
(E) Number the students on the official school roster. Use a table of random numbers to choose 60 students from this roster for the survey.

Answer

37. Die A has three 2's, two 1's and one 0 on its faces. Die B has two 2's and four 3's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. The two dice are tossed. Which of the following represents the probability distribution for the sum of the numbers showing on the two dice?

(A)

x	2	3	4	5
$p(x)$	$\frac{1}{18}$	$\frac{2}{9}$	$\frac{7}{18}$	$\frac{1}{3}$

(B)

x	2	3	4	5
$p(x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

(C)

x	2	3	4	5
$p(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(D)

x	2	3	4	5
$p(x)$	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{7}{9}$	$\frac{1}{18}$

(E)

x	2	3	4	5
$p(x)$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{7}{18}$	$\frac{1}{3}$

Answer

38. The manager of a Mini Mart is running a promotion. For each customer who orders a cup of coffee, he will roll a fair die. If the die shows a six, the customer will be given the coffee for no charge. If the die does not show a six, the customer will pay the usual price for the coffee. One particular customer will order coffee on five separate occasions, and the students in a statistics class will use a simulation make an estimate of the distribution of the number of free cups of coffee this customer will obtain. The students will perform 100 runs of the simulation. When the results of the simulation are shown in a frequency table, which of the following is most likely to result?

(A)

Number of Free Cups of Coffee	Frequency
0	14
1	18
2	17
3	21
4	14
5	16

(B)

Number of Free Cups of Coffee	Frequency
0	0
1	3
2	5
3	14
4	41
5	37

(C)

Number of Free Cups of Coffee	Frequency
0	0
1	0
2	3
3	17
4	39
5	41

(D)

Number of Free Cups of Coffee	Frequency
0	39
1	42
2	14
3	4
4	1
5	0

(E)

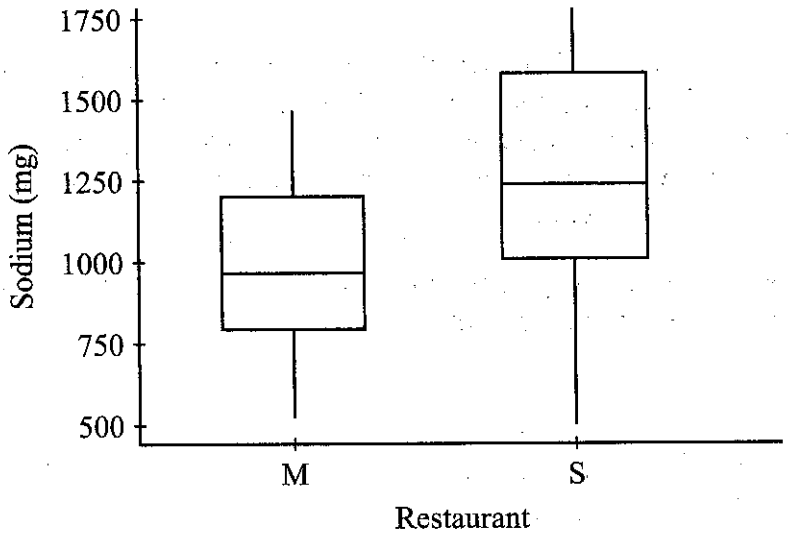
Number of Free Cups of Coffee	Frequency
0	3
1	17
2	36
3	30
4	13
5	1

Answer

SECTION II
Part A
Questions 1–5
Spend about 65 minutes on this part of the exam.
Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

1. Americans typically consume an average of about 3,500 mg of salt per day. The American Heart Association recommends that all Americans reduce consumption to 2,400 mg of salt per day to prevent high blood pressure and other related diseases. Two fast food restaurants each claim to offer healthier diets. We will call these restaurants "M" and "S". A researcher decides to collect data to refute or substantiate the restaurants' claim. The sodium content for all of the "sandwich" options for each restaurant was recorded. Both restaurants had the same number of sandwich options. The boxplots shown below display the sodium content data for both restaurants.



39. For the manager of the Mini Mart in question 38, assume that it costs \$0.35 to produce one cup of coffee. Given that he charges \$1.00 if the die shows a 1, 2, 3, 4 or 5 and \$0 if the die shows a 6, which of the following is closest to the expected value of the profit per cup?

(A) \$0.33
(B) \$0.48
(C) \$0.50
(D) \$0.65
(E) \$0.83

Answer

40. A large sample hypothesis test of the null hypothesis $H_0: \mu = 15$ against the alternative hypothesis $H_a: \mu \neq 15$ results in the test statistic of $z = 1.36$. Assuming the population standard deviation is known, the corresponding p-value is

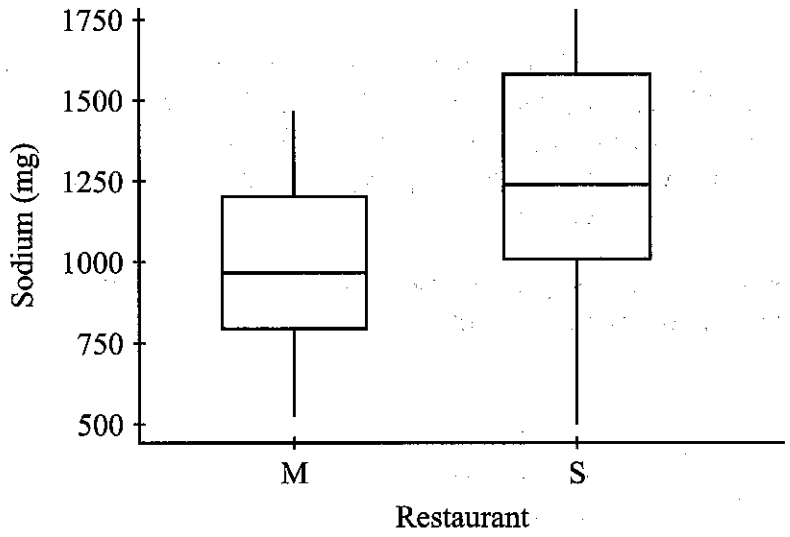
(A) 0.0869
(B) 0.1738
(C) 0.4131
(D) 0.9131
(E) 0.9942

Answer

SECTION II
Part A
Questions 1–5
Spend about 65 minutes on this part of the exam.
Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

1. Americans typically consume an average of about 3,500 mg of salt per day. The American Heart Association recommends that all Americans reduce consumption to 2,400 mg of salt per day to prevent high blood pressure and other related diseases. Two fast food restaurants each claim to offer healthier diets. We will call these restaurants "M" and "S". A researcher decides to collect data to refute or substantiate the restaurants' claim. The sodium content for all of the "sandwich" options for each restaurant was recorded. Both restaurants had the same number of sandwich options. The boxplots shown below display the sodium content data for both restaurants.



Sample Examination Three

SECTION I
Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A survey was conducted to determine the proportion of high school seniors who planned to go to college after high school. Questionnaires were sent to a random sample of high school seniors in the United States during the month of May. The results were: 78% of the sample stated that they planned to attend college after high school. The survey had a margin of error of ±6% at a 95% confidence level. Which conclusion is valid?

   (A) 6% of the population of all high school seniors was surveyed.
   (B) Between 72% and 84% of the sample of seniors stated that they would be going to college after high school.
   (C) The proportion of the entire population of high school seniors who plan to go to college is now known to be between 0.72 and 0.84.
   (D) It is unlikely that the given sample result would have been obtained unless the true population proportion was between 0.72 and 0.84.
   (E) Between 72% and 84% of the population was surveyed.

Answer

(d) Use the answer from part (c) to determine the expected number of donors needed to produce three donors with either type B blood or type O blood.

(e) Blood bank C has the capacity to service 50 donors a day, while blood bank D has a capacity of 200 donors a day. Assuming both blood banks reach their capacity every day, at which blood bank would it be more likely that the percentage of type B or O donors is more than 75% on a given day? Provide statistical evidence to justify your answer.

2. A weight loss program claims that participants will lose on average at least 4 pounds a week. A skeptical consumer group wishes to provide convincing evidence that the program's claim is false. Let the mean weight loss per week in pounds by all participants in the program be represented by μ. Which of the following gives the null and alternative hypotheses that the consumer group should test?

(A) $H_0: \mu = 4$
$H_a: \mu < 4$

(B) $H_0: \mu > 4$
$H_a: \mu < 4$

(C) $H_0: \mu = 4$
$H_a: \mu > 4$

(D) $H_0: \mu = 4$
$H_a: \mu \geq 4$

(E) $H_0: \mu = 4$
$H_a: \mu \neq 4$

Answer

3. A small community college has 3,000 registered students, of whom 40% are full-time and the rest are part-time. Seventy-five percent of the full-time students are education majors and 40% of the part-time students are education majors. If a random sample of 200 students is selected, what is the expected number of students in the sample who are education majors?

(A) 80
(B) 100
(C) 108
(D) 110
(E) 120

Answer

Questions 4 and 5 refer to the information below.

Researchers who study child development found a linear regression model for infant ages that uses age in months to predict height. A sample of 12 babies was randomly selected and the information shown below was generated.

S = 0.256 R-Sq = 68.9% R-Sq(adj) = 69.5%

Variable	N	Mean	Median	TrMean	StDev	SE Mean
age	12	23.500	23.500	23.500	3.610	1.040
height	12	79.850	79.800	79.860	2.302	0.665

4. What is the slope of the least squares regression line?

(A) 0.44
(B) 0.53
(C) 1.08
(D) 1.30
(E) The slope cannot be determined from the given information.

Answer

5. What percentage of the observed variation in height can be explained by the least squares regression line of height on age?

(A) 26%
(B) 29%
(C) 69%
(D) 83%
(E) 95%

Answer

6. Below is a probability distribution showing the year after purchase in which a certain brand of computer first fails:

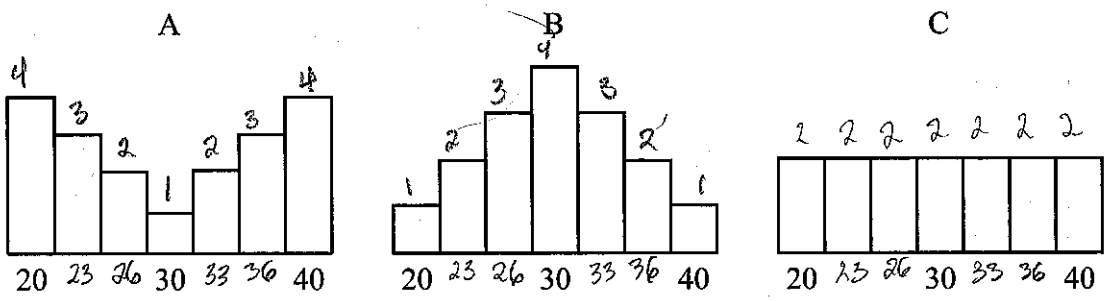
Year	1	2	3	4	5	6 or more
Probability	.12	.02	.05	.08	.11	0.62

(This means, for example, that a computer that experiences its first failure in year 1 has failed in *less than* one year.) A small business buys three of these computers. Assuming that the three computers function independently, what is the probability that exactly two of the three computers will fail in less than three years?

(A) $(0.19)^2(0.81)$
(B) $3(0.19)^2(0.81)$
(C) $(0.14)^2(0.86)$
(D) $3(0.14)^2(0.86)$
(E) $3(0.12)^2(0.88) + 3(0.02)^2(0.98)$

Answer

Questions 7 and 8 refer to the graphs below.



7. Which distribution has the smallest standard deviation?

(A) A
(B) B
(C) C
(D) The answer cannot be determined from the graphs.
(E) They all have the same standard deviation.

Answer

8. In which distribution are the mean and the median equal?

(A) A only
(B) B only
(C) C only
(D) A and B only
(E) A, B and C

Answer

9. The results of a simple random sample of size 30 will be used to test the null hypothesis $\beta = 0$ against the alternative hypothesis $\beta \neq 0$, where β is the slope of a population regression line. The resulting test statistic is $t = 2.046$. Concerning the p-value for the test, which of the following is true?

(A) $0.01 < p < 0.02$
(B) $0.02 < p < 0.03$
(C) $0.03 < p < 0.04$
(D) $0.04 < p < 0.05$
(E) $0.05 < p < 0.1$

Answer

10. An inspection procedure at a manufacturing plant involves picking three items at random from a large batch, and then accepting the whole batch if at least two of the three items are in perfect condition. If in reality 80% of the items in the whole batch are perfect, what is the probability that the batch will be accepted?

(A) 0.512
(B) 0.560
(C) 0.640
(D) 0.896
(E) 0.992

Answer

11. Which of the following statements about simple random samples is true?

I. A simple random sample is any sample chosen in such a way that each element of the population has the same chance of being selected.
II. A simple random sample of size n is a sample chosen in such a way that every set of n elements from the population has the same chance of forming the sample.
III. A sample of 4 students is selected from a class of 10 girls and 10 boys by randomly selecting 2 of the 10 girls and 2 of the 10 boys. This is an example of a simple random sample.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II and III

Answer

12. The level of significance of a hypothesis test is always equal to the

(A) probability that a Type II error occurs
(B) probability that a Type I error occurs
(C) probability that a Type I error does not occur
(D) probability that a Type II error does not occur
(E) p-value of the test

Answer

13. A study was performed to determine whether there is an association between the method of teaching – lecture or discussion – and the subject matter of the class that was being taught. Nine art classes, 11 math classes and 10 science classes were observed, and the type of teaching was recorded for each class. The following results were obtained.

	Discussion	Lecture	
Art	1	8	9
Math	8	3	11
Science	6	4	10

Which of the following best describes the relationship between method of teaching and type of class?

(A) There appears to be no association since the number of discussion classes and the number of lecture classes were exactly the same.
(B) No association can be determined since the numbers of art, math and science classes were not exactly the same.
(C) There appears to be an association since the art classes were less likely to use discussion than either the math or the science classes.
(D) There appears to be an association since both the number of math and the number of science classes were greater than the number of arts classes.
(E) No information regarding this association can be gained from the data given in the table.

7

Answer

14. A researcher interested in the age at which women are having their first child surveyed a simple random sample of 250 women who had at least one child. He found that the distribution of the age at which women have their first child was approximately normal with a mean of 22.3 and a standard deviation of 5.4. According to the empirical rule, approximately 95% of the women have their first child between the ages of

(A) 11.5 years and 33.1 years
(B) 16.9 years and 27.7 years.
(C) 16.9 years and 33.1 years
(D) 20.0 years and 24.6 years
(E) 20.0 years and 36.9 years

Answer

15. At a certain high school, 1/7 of all students take AP Statistics, 1/4 of all students play football, and these two events are independent. If a student is chosen at random from the school, what is the probability that the student is involved in at least one of these two pursuits?

(A) 7/28
(B) 9/28
(C) 11/28
(D) 13/28
(E) 5/14

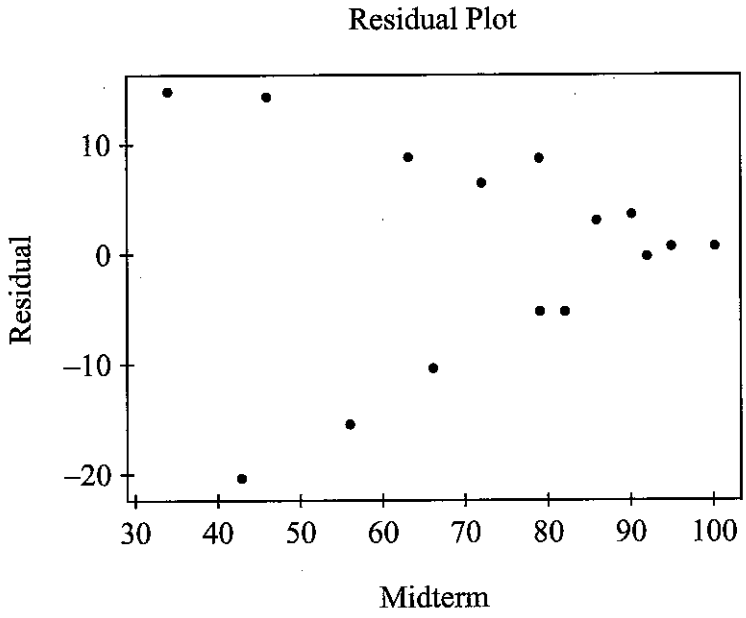
Answer

16. Early studies of probability were conducted by the Italian mathematician Girolamo Cardano (1501-1576). One of the many dice games that Cardano studied was played with six 6-sided dice. Each of these six dice had five blank faces and one face with a number, and each of the numbers 1 through 6 appeared on exactly one of the six dice. All 6 dice were rolled at once, and the payoff to the gambler was based on the sum of the numbers showing on the up faces. What is the the expected value of the sum obtained by rolling all 6 dice?

(A) 3
(B) 3.5
(C) 6
(D) 36
(E) 42

Answer

17. The residual plot below came from a linear regression using data on grade at midterm against grade on final exam. Which conclusion could be reached by analyzing the residual plot?



(A) An exponential curve should be used to predict the final grade from the midterm grade.
(B) A parabola should be used to predict the final grade from the midterm grade.
(C) A cubic curve should be used to predict the final grade from the midterm grade.
(D) There is evidence of a greater variation from the regression line for students with low midterm scores than for students with high midterm scores.
(E) Students did better on the final exam than they did on the midterm.

Answer

18. The heights of American men aged 18 to 24 are approximately normal with a mean of 68 inches and a standard deviation of 2.5 inches. About 20% of these men are taller than

(A) 66 inches
(B) 68 inches
(C) 70 inches
(D) 72 inches
(E) 74 inches

Answer

19. Which of the following will most likely approximate a uniform distribution?

(A) The heights of students at a particular high school
(B) The weights of students at a particular high school
(C) The SAT scores of seniors at a particular high school
(D) The IQ scores of students at a particular high school
(E) The ages of students at a particular high school

Answer

20. At a certain high school, a simple random sample of fifty-two 11^{th} and 12^{th} graders was asked about their political affiliations. The results are summarized in the following two-way table. If a χ^2 test were performed using these data, what would be the appropriate number of degrees of freedom?

	11^{th} Grade	12^{th} Grade
Republican	11	5
Democrat	10	15
Independent	5	6

(A) 1
(B) 2
(C) 3
(D) 6
(E) 25

Answer

21. The scores on a standardized test designed to measure math anxiety are normally distributed with a mean of 100 and a standard deviation of 10 for a population of first year college students. Which of the following observations would be considered an outlier?

(A) 90
(B) 100
(C) 150
(D) All of the above
(E) None of A, B, or C would be an outlier.

Answer

22. A six-sided die has one 1, two 3's, two 4's and one 6. The mean and the standard deviation of the score on this die are 3.5 and 1.5, respectively. What are the mean and the standard deviation of the total score when this die is rolled five times?

(A) Mean = 3.5, standard deviation = 0.67
(B) Mean = 3.5, standard deviation = 0.95
(C) Mean =17.5, standard deviation = 2.74
(D) Mean = 17.5, standard deviation = 3.35
(E) Mean = 17.5, standard deviation = 7.5

Answer

23. The heights of 17-year-old boys are normally distributed with a standard deviation of 2.25 inches. How does the height of a boy at the 10^{th} percentile compare with the mean height of 17-year-old boys?

(A) Approximately 1.89 inches below the mean
(B) Approximately 2.5 inches below the mean
(C) Approximately 2.88 inches below the mean
(D) Approximately 3.7 inches below the mean
(E) Approximately 4.5 inches below the mean

Answer

24. Which of the following statements about a least squares linear regression model is true?

I. The sum of the residuals is always zero.
II. If $r^2 = 0$, the regression line is horizontal.
III. A random pattern in the residual plot is an indication that a nonlinear model will fit the data better than the straight-line regression model.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

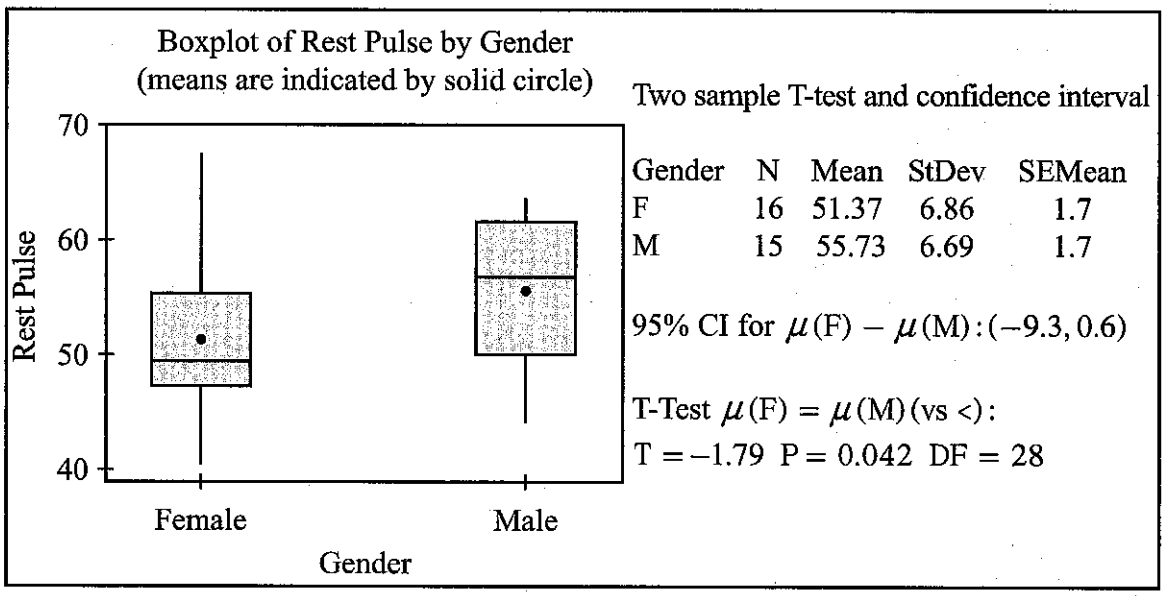
Answer

25. Using a random integer generator, Natalie generates 200 samples of 30 random integers between 0 and 4, inclusive. She records the total number of 0's and 1's in each of the 200 samples. Which of the following is the distribution she has simulated?

(A) A sampling distribution of sample proportions with $n = 200, p = 0.4$
(B) A sampling distribution of sample proportions with $n = 30, p = 0.2$
(C) A binomial distribution with $n = 5, p = 0.2$
(D) A binomial distribution with $n = 30, p = 0.4$
(E) A binomial distribution with $n = 200, p = 0.4$

Answer

26. A study attempted to compare the at rest pulse rates of men and women between the ages of 35 and 60. Two independent random samples were taken, one from the population of females and the other from the population of males, and the at rest pulse rates of the men and women in the samples were measured. The computer output shown below gives the results for a test of $H_0: \mu_{females} = \mu_{males}$ *versus* $H_a: \mu_{females} < \mu_{males}$.



Two sample T-test and confidence interval

Gender	N	Mean	StDev	SEMean
F	16	51.37	6.86	1.7
M	15	55.73	6.69	1.7

95% CI for $\mu(F) - \mu(M)$: $(-9.3, 0.6)$

T-Test $\mu(F) = \mu(M)$ (vs <):
$T = -1.79$ $P = 0.042$ $DF = 28$

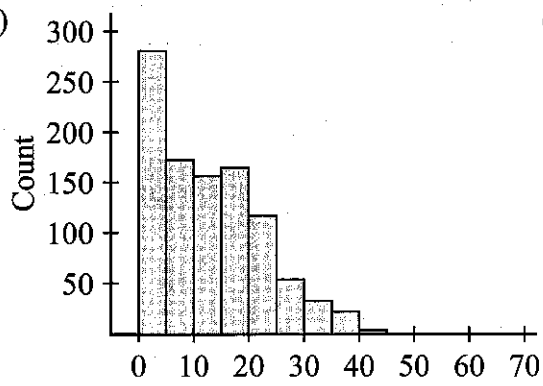
Which of the following is a correct conclusion from the analysis shown above?

(A) The mean at rest pulse rate for the females is not significantly lower than the mean at rest pulse rate of the males at the 0.05 level of significance.
(B) The mean at rest pulse rate for the females is significantly lower than the mean at rest pulse rate of the males at the 0.05 level of significance.
(C) The mean at rest pulse rate for the females is significantly lower than the mean at rest pulse rate of the males at the 0.01 level of significance.
(D) The mean at rest pulse rate for the females is significantly higher than the mean at rest pulse rate of the males at the 0.05 level of significance.
(E) There is evidence at the 0.05 level of significance that the mean at rest pulse of men is different from the mean at rest pulse of women.

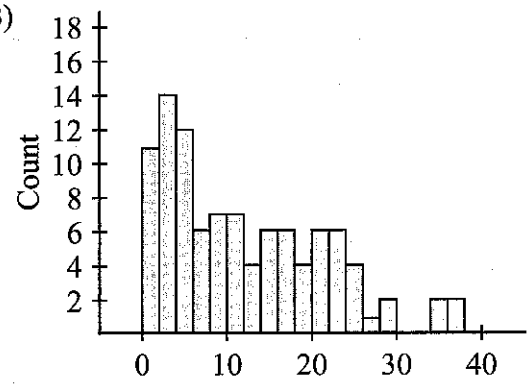
Answer

27. A population distribution is skewed right with a mean of 12.3 and a standard deviation of 9.6. A student takes 1000 random samples of size 100 from this distribution and graphs all 1000 of the sample means using a histogram. Which of the following is most likely for this simulated sampling distribution of the mean?

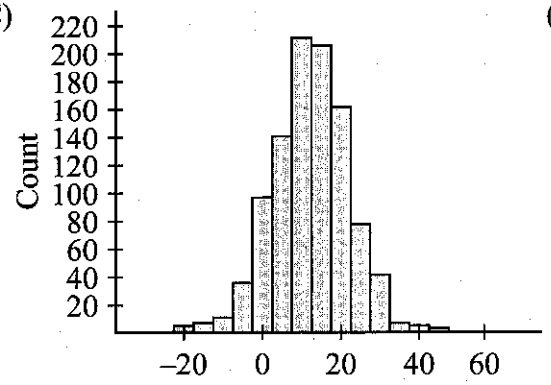
(A)



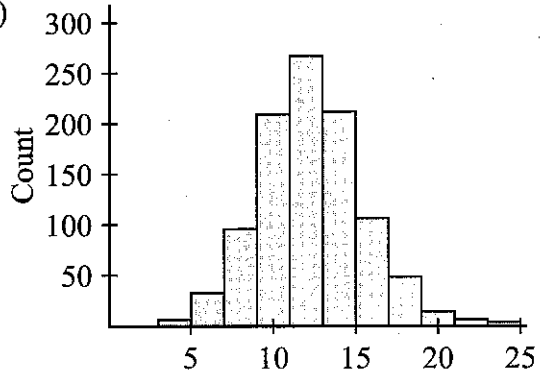
(B)



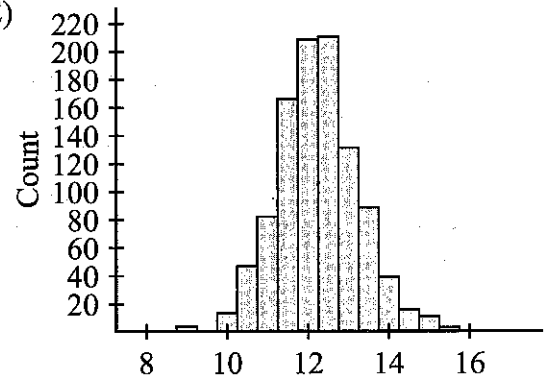
(C)



(D)



(E)



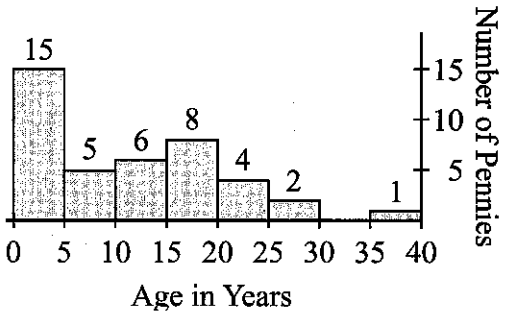
Answer

28. A two-tailed hypothesis test is conducted to determine if there is a difference between two population means. The population standard deviations are unknown. The test reports a p-value of .074. Using the same data, a confidence interval for $\mu_1 - \mu_2$ is calculated. Of the following, which is the lowest level of confidence for which the interval will contain 0?

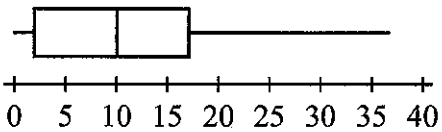
(A) 90%
(B) 92%
(C) 93%
(D) 95%
(E) 98%

Answer

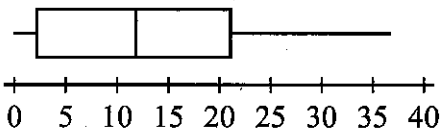
29. The histogram below displays the ages of a random sample of 41 pennies drawn from a population of 1,000 pennies. Which of the following boxplots could represent this distribution of ages?



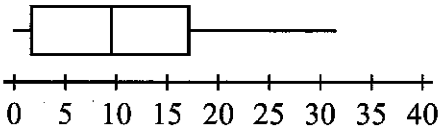
(A)



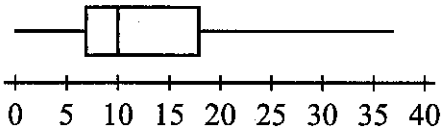
(B)



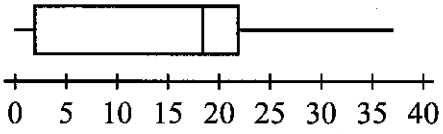
(C)



(D)



(E)



Answer

30. The massed marching bands of the Middle States region have gathered for their annual gala. The musicians present consist entirely of brass players, woodwind players, and percussionists. Certain changes in procedure have been implemented this year, and so, in order to gather opinion on these changes, the organizers decide to take a sample of the musicians and to interview the people selected. The organizers suspect that there might be differences of opinion between the players of different types of instrument. Therefore, in order to form the sample, the organizers select a random sample of brass players, a random sample of woodwind players, and a random sample of percussionists. This is an example of a

(A) simple random sample
(B) cluster sample
(C) stratified random sample
(D) systematic sample
(E) convenience sample

Answer

31. The table below shows the heights in inches of 9 randomly selected married couples. Which of the following significance tests could be used to determine if, on average, men are taller than the women they marry? (You may assume that all conditions for inference are met.)

Couple	1	2	3	4	5	6	7	8	9
Husband	72	70	74	65	71	70	71	66	67
Wife	69	62	65	67	68	66	68	64	67

(A) Two sample z-test
(B) Two sample t-test
(C) Paired t-test
(D) Chi-square test for homogeneity
(E) t-test for the slope of the regression line

Answer

32. The principal of a school wants to know if the method of instruction (lecture *vs.* lecture and exploratory activities) influences how well classes learn. Using two statistics classes taught by the same teacher, the principal flips a coin to decide which class will be taught using lecture only. The other class is to be taught by both lecture and group exploratory activities. Which statement does NOT explain why the design of this experiment is flawed.

(A) Since the same teacher teaches both classes, classes must meet at different times of the day. Influences of time of day may be confounded with teaching method.
(B) There is insufficient replication in the experiment.
(C) It is possible that the two classes differ in ability. Therefore, ability could be confounded with teaching method.
(D) There is no control group in the experiment.
(E) Since the principal is using only statistics classes taught by the same teacher, the principal cannot generalize the results of the study to all classes and teachers in the school.

Answer

33. If a fair coin is tossed five times and comes up heads all five times, then the probability of a tail on the sixth toss is

(A) $\frac{1}{32}$
(B) $\frac{3}{32}$
(C) Slightly less than $\frac{1}{2}$
(D) $\frac{1}{2}$
(E) More than $\frac{1}{2}$

Answer

34. Suppose A and B are events with the given probabilities: $P(A) = \frac{3}{4}$, and $P(B) = \frac{1}{2}$, and $P(B \text{ given } A) = \frac{1}{2}$. Which of the following conclusions can NOT be drawn from the given information?

(A) $P(A \text{ and } B) = \frac{3}{8}$

(B) $P(A \text{ or } B) = \frac{7}{8}$

(C) $P(A \text{ given } B) = \frac{1}{2}$

(D) A and B are not mutually exclusive events.

(E) A and B are independent events.

Answer

35. A farmer has a 100-acre square field that has been planted with corn. The field has been divided into a 10 by 10 grid of plots, making 100 square plots, each of area 1 acre. The east end of his field is next to a forest. Because of the shade the trees offer, this side of the field is the least productive. On the west end of the field is a river. The closer the plots of land are to the river, the more fertile the soil is and the more productive the land. He wished to estimate the yield of the whole field by taking a sample of ten of the one hundred plots. Of the methods given below, which is likely to give the most accurate estimate of the yield of his field?

(A) He should randomly select 5 plots from the ten 1-acre plots closest to the river and 5 plots from the ten 1-acre plots furthest from the river.

(B) He should number all the plots of land from 1 to 100 and use a random number generator to select the ten plots.

(C) He should divide the field into 10 rows, each row starting on the east and ending on the west end of the field. He should number each of the 1-acre plots in each of the ten rows from 1-10. Then using a random number generator select one 1-acre field from each of the east/west rows.

(D) He should divide the field into 10 rows, each row starting on the east and ending on the west end of the field. He should number the rows from 1 to 10. Then using a random number generator, he should select one of the east/west rows and sample all ten 1-acre fields in that row.

(E) He should divide the field into 10 columns, each column starting on the north and ending on the south end of the field. He should number each of the 1-acre plots in each of the ten columns from 1-10. Then using a random number generator, select one 1-acre field from each of the north/south columns.

Answer

36. Consider the set of n points, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. Let the mean of the x-values be $\bar{x}$ and the mean of the y-values be $\bar{y}$. Suppose, now, that an attempt is made to fit a straight line to the set of points, and for each x-value, x_i, the value of y predicted by the line is $\hat{y}_i$. If the line that is fitted to the set of points is, in fact, the least squares regression line, which of the following is being minimized?

(A) $\hat{y}_i$
(B) $x_i - \bar{x}$
(C) $\sum(y_i - \bar{y})^2$
(D) $\sum(x_i - \bar{x})^2$
(E) $\sum(y_i - \hat{y}_i)^2$

Answer

37. A student wishes to know if a majority of students at his school approve of changing the food selections in the cafeteria. He takes a simple random sample of 100 students from a list of the 4,000 in the entire school. He then asks each student in the sample whether he/she is in favor of changing the food selections in the cafeteria. If 65 of the students are in favor of changing the selections, the value of the test statistic is

(A) $z = \dfrac{(0.65 - 0.5)}{\sqrt{\dfrac{(0.65)(0.35)}{100}}}$
(B) $z = \dfrac{(0.65 - 0.5)}{\sqrt{\dfrac{(0.5)(0.5)}{100}}}$
(C) $z = \dfrac{(0.65 - 0.5)}{\sqrt{\dfrac{(0.65)(0.35)}{4000}}}$
(D) $z = \dfrac{(0.65 - 0.5)}{\sqrt{\dfrac{(0.5)(0.35)}{100}}}$
(E) $z = \dfrac{(0.65 - 0.5)}{\sqrt{\dfrac{(0.5)(0.5)}{4000}}}$

Answer

38. A hypothesis test is to be conducted using the results from a simple random sample. Which of the following is true?

(A) The power of the test increases as α, the probability of making a Type I error, decreases.
(B) The power of the test decreases as the sample size, n, increases.
(C) The power of the test increases as α increases.
(D) The power of the test is the probability that the null hypothesis is false.
(E) The power of the test increases as β, the probability of making a Type II error, increases.

Answer

39. Suppose that, for a particular set of data, the regression line, $\hat{y} = 3x + b$, passes through the point (2, 5). If $\bar{x}$ and $\bar{y}$ are the sample means of the x- and y-values respectively, then $\bar{y} =$

(A) $\bar{x}$
(B) $\bar{x} - 2$
(C) $\bar{x} + 5$
(D) $3\bar{x}$
(E) $3\bar{x} - 1$

Answer

40. Temperature data were recorded in degrees Celsius for a certain community in England. The data were converted to degrees Fahrenheit using the formula, Degrees Fahrenheit $= \frac{9}{5}$ (Degrees Celsius) $+ 32$. Which of the following is NOT true about the transformed data?

(A) The mean of the transformed data in degrees Fahrenheit equals $\frac{9}{5}$ (mean in degrees Celsius) $+ 32$.

(B) The median of the transformed data in degrees Fahrenheit equals $\frac{9}{5}$ (median in degrees Celsius) $+ 32$.

(C) The variance of the transformed data in degrees Fahrenheit equals $\frac{9}{5}$ (variance in degrees Celsius).

(D) The standard deviation of the transformed data in degrees Fahrenheit equals $\frac{9}{5}$ (standard deviation in degrees Celsius).

(E) The range of the transformed data in degrees Fahrenheit equals $\frac{9}{5}$ (range in degrees Celsius).

Answer

Sample Examination Four

SECTION I
Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. There are 24 people who work in a particular office. Their travel times (in minutes) from home to work are listed below.

   30 35 35 39 40 44 47 48 49 50 50 50 54 55 56 56 57 58 59 59 62 62 63 66

   Which of the following is closest to the 20th percentile travel time (in minutes) for these 24 people?

   (A) 30
   (B) 40
   (C) 50
   (D) 60
   (E) 70

Answer

2. A young basketball player estimates that he is successful on one in every three free throws. He wishes to simulate a sequence of free throws using a table of random digits, and he is willing to assume that the results of the throws are independent of each other. Of the following, which would be the best strategy to use?

   (A) Take one digit at a time, and have 0-2 represent success and 3-9 represent failure
   (B) Take one digit at a time, and have 0-2 represent success, 3-8 represent failure, and ignore 9's
   (C) Take two digits at a time, and have 00-32 represent success and 33-99 represent failure
   (D) Take two digits at a time, and have 00-29 represent success and 30-99 represent failure
   (E) Take three digits at a time, and have 000-333 represent success and 334-999 represent failure

Answer

3. The director of a company wants to know the mean salary of all the company's employees. The most effective way of determining this is to

   (A) record the salaries of a simple random sample of employees
   (B) record the salaries of a stratified random sample of employees
   (C) record the salaries of a cluster sample of employees
   (D) record the salaries of a convenience sample of employees
   (E) consult computer records regarding the salaries of all employees

Answer

4. Suppose that a study of 500 randomly chosen teenagers established that those who had regularly played video games over the previous year tended to have a greater susceptibility to apathy (lack of motivation) than those who had not regularly played video games. Of the following, which would be the best way to establish whether regular playing of video games by teenagers causes apathy?

   (A) Repeat the study, this time stratifying by gender
   (B) Repeat the study, this time making sure that all ages between 13 and 19, inclusive, are covered adequately
   (C) Repeat the study, this time increasing the sample size to 1000 teenagers
   (D) Repeat the study, this time making sure that all types of video game are used
   (E) Perform a study using 50 teenagers. Randomly choose which 25 will play video games regularly and which 25 will not, and observe any connection between video game playing and apathy

Answer

5. In a large company, it has been assumed for several years that approximately 65% of the employees regularly eat the lunches provided by the company. However, it is now suspected that this figure has increased. In order to test this, a random sample of 50 employees is selected, and it is found that 36 of them regularly eat the lunches provided. If a significance test is to be performed, and the proportion of all employees who regularly eat the meals is to be denoted by p, what are the null and alternative hypotheses that should be used?

   (A) $H_0: p = 0.72$; $H_a: p < 0.72$
   (B) $H_0: p = 0.72$; $H_a: p > 0.72$
   (C) $H_0: p = 0.65$; $H_a: p = 0.72$
   (D) $H_0: p = 0.65$; $H_a: p < 0.65$
   (E) $H_0: p = 0.65$; $H_a: p > 0.65$

Answer

6. The sugar level and the protein level were measured for each animal in a sample of bulls. A scatterplot was drawn with sugar level plotted on the horizontal (x) axis and protein level plotted on the vertical (y) axis. The coefficient of determination, r^2, between these two variables was found to be 0.81. Of the following, which is the best interpretation of this value of r^2?

(A) 81% of the variation in protein levels can be explained by the least squares regression line relating protein level and sugar level.
(B) 81% of the variation in protein levels and sugar levels can be explained by the least squares regression line relating protein level and sugar level.
(C) 81% of the protein levels can be explained by the least squares regression line relating protein level and sugar level.
(D) 81% of the sugar levels can be explained by the least squares regression line relating protein level and sugar level.
(E) 81% of the relationship between sugar level and protein level can be explained by the least squares regression line.

Answer

7. In a study, 12 subjects were required to perform a task with their dominant hand and the same task with their non-dominant hand. The following results were obtained:

Subject	1	2	3	4	5	6	7	8	9	10	11	12
Time with Dominant Hand (seconds)	30	20	24	17	26	27	19	22	18	24	22	33
Time with non-Dominant Hand (seconds)	34	23	26	16	28	27	21	25	20	28	26	40

Assuming that the conditions for inference are met, which of the following significance tests could legitimately be used to analyze this dataset?

I. Two-sample t-test
II. Paired t-test
III. t-test for the slope of the regression line

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

Answer

8. A fair six-sided die with faces numbered 1 through 6 is rolled 5 times. What is the probability that the number of sixes rolled is either 1 or 2?

(A) 0.093
(B) 0.161
(C) 0.482
(D) 0.563
(E) 0.965

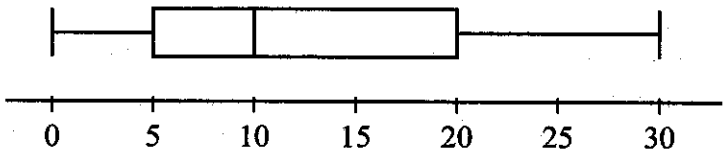
Answer

9. A statistician takes a simple random sample of size 15 from a very large population whose standard deviation is unknown. The statistician's intention is to perform a one-sample *t*-test for the mean. A boxplot of the sample values is drawn, and it is found that the distribution of the sample values is roughly symmetrical and that there are no outliers. The statistician is now justified in using the *t*-test because

(A) the boxplot has shown that the sample is large enough for use of the *t*-distribution
(B) it is known that the sample is *t*-distributed
(C) it is known that the population is *t*-distributed
(D) it is known that the sample is normally distributed
(E) it is feasible that the population is normally distributed

Answer

10. A set of 23 measurements is summarized by the following boxplot.



Which of the following could be the mean of the 23 measurements?

(A) 7
(B) 9
(C) 12
(D) 22
(E) 27

Answer

11. A study is carried out to compare the effectiveness of two types of fertilizer, A and B. 260 seeds of a particular plant are randomly assigned to the two types of fertilizer. Of the 120 seeds assigned to fertilizer A, 83 are found to germinate, and of the 140 seeds assigned to fertilizer B, 78 are found to germinate. Which of the following is a 99% confidence interval for $p_A - p_B$, the difference between the two proportions of all seeds germinating for the two types of fertilizer?

(A) $\left(\frac{83}{120} - \frac{78}{140}\right) \pm 2.326\sqrt{\frac{(83/120)(37/120)}{120} + \frac{(78/140)(62/140)}{140}}$

(B) $\left(\frac{83}{120} - \frac{78}{140}\right) \pm 2.326\sqrt{\frac{(83/120)(78/140)}{120} + \frac{(37/120)(62/140)}{140}}$

(C) $\left(\frac{83}{120} - \frac{78}{140}\right) \pm 2.326\sqrt{\frac{161}{120} \cdot \frac{99}{260}\left(\frac{1}{120} + \frac{1}{140}\right)}$

(D) $\left(\frac{83}{120} - \frac{78}{140}\right) \pm 2.576\sqrt{\frac{(83/120)(37/120)}{120} + \frac{(78/140)(62/140)}{140}}$

(E) $\left(\frac{83}{120} - \frac{78}{140}\right) \pm 2.576\sqrt{\frac{161}{260} \cdot \frac{99}{260}\left(\frac{1}{120} + \frac{1}{140}\right)}$

Answer

12. In a particular state, a polling organization wishes to find out whether there is a difference in political preference between the over-forties and the under-forties. A random sample of people over the age of forty will be taken, and a random sample of people under the age of forty will be taken. Each person will be asked whether he/she supports the democrats, the republicans, or some other party. Of the following, which would be the most appropriate significance test to analyze the results of the study? You may assume that the conditions for inference are met.

(A) One-proportion z-test
(B) Two-sample t-test for the difference of two means
(C) Paired t-test for the difference of two means
(D) Chi-square test for goodness of fit
(E) Chi-square test for homogeneity

Answer

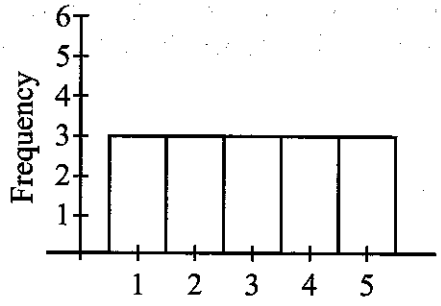
13. A marketing company is interested in finding out whether telephone salespeople would perform better if they did relaxation or stretching exercises before they start their work. A set of salespeople will be randomly assigned to three groups. For a few minutes before they start work, the salespeople in the first group will do relaxation exercises, the salespeople in the second group will do stretching exercises, and the salespeople in the third group will be given some free time. The sales performances of the people in the three groups will then be compared. In the context of this experiment, which of the following is an instance of replication?

(A) Using a set of salespeople that represents the population of interest
(B) Making sure that the sales portion of the experience is the same for the three groups
(C) Making sure that the groups contain roughly equal numbers of experienced salespeople and inexperienced salespeople
(D) Using a large enough number of salespeople so that any differences between the groups (in terms of the people assigned) become negligible
(E) Getting a result for the experiment that is correct for the population of interest

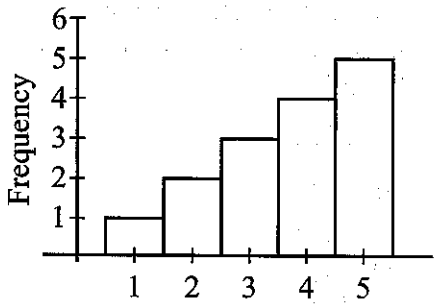
Answer

14. Of the distributions represented by the histograms below, which has the smallest standard deviation?

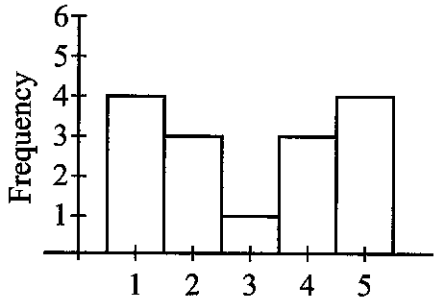
(A)



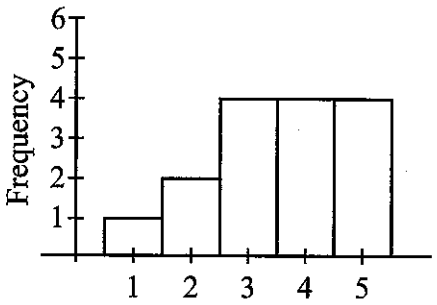
(B)



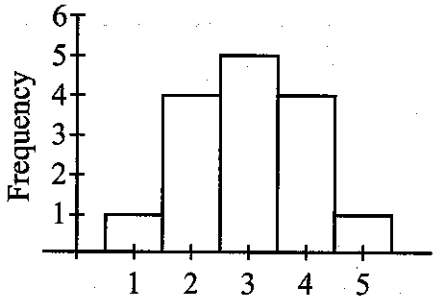
(C)



(D)



(E)



Answer

15. A college offers two courses in calculus: Calculus 1 and Calculus 2. Last semester, 60.9% of calculus students took Calculus 1 and the rest took Calculus 2. (No student took both courses.) Of the students who took Calculus 1, 28.0% received A's, and of the students who took Calculus 2, 38.1% received A's. If a student is selected at random from those who received A's in calculus, what is the probability that the student took Calculus 1?

(A) 0.170
(B) 0.280
(C) 0.534
(D) 0.609
(E) 0.720

Answer

16. A large number of students at a high school take an Algebra II course that emphasizes calculator use. The chairman of the math department is concerned that students' mental arithmetic skills might be declining through lack of practice during the course. He selects a random sample of students who take the course and gives them a mental arithmetic test (scored out of 30) at the beginning of the course (PreTest) and a comparable test at the end of the course (PostTest). The computer output shown below gives the results of a test of $H_0: \mu_D = 0$ *versus* $H_a: \mu_D < 0$, where D denotes the difference in scores (PostTest − PreTest).

Matched Pairs
Difference: PostTest - PreTest

PreTest Mean	28.4286	t-Ratio	-2.065591
PostTest Mean	27.2857	DF	13
Mean Difference	-1.14286	Prob < t	0.0297
Std Error	0.55328		
N	14		
Correlation	0.8251		

Assuming that the conditions for inference are met, which of the following is the best conclusion that can be drawn from the analysis?

(A) There is evidence at the 5% level (but not the 1% level) that on average the students' mental arithmetic abilities have increased.
(B) There is evidence at the 1% level that on average the students' mental arithmetic abilities have increased.
(C) There is evidence at the 5% level that there has been a change in the students' average mental arithmetic ability.
(D) There is evidence at the 5% level (but not the 1% level) that on average the students' mental arithmetic abilities have decreased.
(E) There is evidence at the 1% level that on average the students' mental arithmetic abilities have decreased.

Answer

17. A very large population has mean μ and standard deviation σ. A random sample of size n (where n is at least 2) is to be taken from the population. The sample mean will be denoted by $\bar{x}$. Which of the following is NOT known to be true?

(A) The mean of all the possible values of $\bar{x}$ is μ.

(B) The standard deviation of the sample is $\frac{\sigma}{\sqrt{n}}$.

(C) If n is large, the distribution of all possible values of $\bar{x}$ is approximately normal.

(D) The larger the sample, the smaller the standard deviation of all the possible values of $\bar{x}$.

(E) The standard deviation of all the possible values of $\bar{x}$ is less than σ.

Answer

18. The scores on a math test are said to be negatively skewed (skewed to the left). Which of the following is the best interpretation of this statement?

(A) The spread of the scores below the median is greater than the spread of the scores above the median.

(B) The spread of the scores above the median is greater than the spread of the scores below the median.

(C) When a histogram of the scores is drawn, the highest point in the histogram is towards the left of the distribution.

(D) There are more low scores than high scores.

(E) There are more high scores than low scores.

Answer

19. The students in a statistics class participated in an activity concerning the value of a particular population parameter. An observational study was conducted, and then the results of the study were analyzed by means of a significance test involving the null hypothesis H_0. After the analysis was completed, the teacher, who knew the true value of the parameter, announced (correctly) that a Type II error had occurred.

    Which of the following can NOT be concluded from the information given?

    (A) The study was conducted poorly.
    (B) The significance test failed to reach a correct conclusion.
    (C) H_0 was false.
    (D) H_0 was not rejected.
    (E) A Type I error did not occur.

Answer

20. A study is conducted to compare the effects of aerobic and anaerobic exercise on the production of a particular growth hormone. Forty volunteer teenagers with slowed growth are randomly assigned to two groups. The teenagers in the first group are supervised doing aerobic exercise and the teenagers in the second group are supervised doing anaerobic exercise. The levels of the growth hormone in the volunteers are measured at the beginning and at the end of the study. Is the study an experiment or an observational study?

    (A) It is an experiment because the teenagers used did not form a random sample.
    (B) It is an experiment because the teenagers were made to exercise in particular ways.
    (C) It is an experiment because the hormone levels were measured before and after the exercise.
    (D) It is an observational study because there is no control group.
    (E) It is an observational study because the teenagers were observed exercising.

Answer

21. There is a standard weight in ounces for squash balls. A machine produces squash balls whose weights are normally distributed. The machine is adjusted so that the mean weight of the balls produced is the standard weight. To the nearest one thousandth, what should be the standard deviation of the weights of the balls produced by the machine so that 98% of the balls have weights that are within 0.2 ounce of the standard weight?

(A) 0.086 ounce
(B) 0.097 ounce
(C) 0.372 ounce
(D) 0.408 ounce
(E) 0.602 ounce

Answer

22. For a random sample of 20 salamanders, the slope of the regression line for predicting weights from lengths is found to be 4.169, and the standard error of this estimate is found to be 2.142. When performing a test of $H_0: \beta = 0$ against $H_a: \beta \neq 0$, where β is the slope of the regression line for the population of salamanders, the t-value is

(A) 0.435
(B) 0.514
(C) 1.946
(D) 8.258
(E) 8.704

Answer

23. In the context of an observational study, which of the following is most appropriate as an instance of bias?

(A) Getting a sample that over-emphasizes some characteristic of the population that is relevant to the study
(B) Designing a sampling method that is likely to result in a sample that will over-emphasize some characteristic of the population that is relevant to the study
(C) Wrongly interpreting an association between two characteristics as the causation of one by the other
(D) Getting a false conclusion to the study
(E) Revealing a prejudiced attitude amongst the population

Answer

24. A customer at a supermarket buys 11 items. The prices (in dollars) of the items are summarized in the table below.

Mean	3.4572727
Std Dev	1.6129916
Std Err Mean	0.4863353
upper 95% Mean	4.5408952
lower 95% Mean	2.3736502
N	11
Min	1.4900
Q1	2.5000
Median	2.9900
Q3	4.6600
Max	7.1300

The z-score for one of the items is -0.290. What is the price of this item?

(A) \$2.84
(B) \$2.99
(C) \$3.32
(D) \$3.37
(E) \$3.93

Answer

25. A random sample of size 15 is taken from a population, and a 95% confidence interval for the population mean is calculated, from the sample data, to be (64.06, 66.96). Of the following, which gives the best interpretation of 95% confidence in this context?

(A) If many random samples of size 15 from this population are taken, then 95% of the time the population mean will be within the interval (64.06, 66.96).
(B) If many random samples of size 15 from this population are taken, then 95% of the time the sample mean will be within the interval (64.06, 66.96).
(C) If many random samples of size 15 from this population are taken and the confidence intervals are calculated in the same way, then 95% of the confidence intervals will contain the population mean.
(D) If many random samples of size 15 from this population are taken and the confidence intervals are calculated in the same way, then 95% of the confidence intervals will contain the sample mean.
(E) 95% of the population measurements lie within the interval (64.06, 66.96).

Answer

26. Terry is standing in line waiting for a snack. He will receive one serving of chips and one serving of salsa. The amount of chips (in grams) per serving is a random variable with standard deviation 3.7. The amount of salsa (in grams) per serving is a random variable with standard deviation 2.8. The chips contain 5 calories per gram and the salsa contains 0.5 calories per gram. The amounts of the two snacks that Terry receives can be considered to be independent. What is the standard deviation (in grams) of the total number of calories Terry receives?

(A) 4.46
(B) 8.51
(C) 18.55
(D) 19.90
(E) 344.21

Answer

27. A spelling quiz is taken by 12 girls and 20 boys. The mean score for the girls is 8.25 and the mean score for the boys is 7.3. Of the following, which is closest to the mean score for all of the students?

(A) 7.66
(B) 7.71
(C) 7.76
(D) 7.78
(E) 7.89

Answer

28. Four regular (fair) six-sided dice are rolled. To the nearest one-thousandth, what is the probability that the largest of the four scores is a six?

(A) 0.096
(B) 0.386
(C) 0.518
(D) 0.668
(E) 0.783

Answer

29. The manager of a clothing store selects a random sample of 30 items of clothing that have been returned to the store. For each item in the sample the manager determines whether the item has been worn (W) or has not been worn (N). The following results are obtained.

W N N N N W W N N W W W W W W
W W N N N N W W N W W W N W W

A significance test is to be conducted in order to determine whether these results provide convincing evidence that a majority of items returned to the store have been worn. Which of the following is closest to the p-value for the test?

(A) 0.032
(B) 0.137
(C) 0.264
(D) 0.273
(E) 0.282

Answer

30. Max randomly selects a sample of five students from his math class, and asks each student in the sample how many phones he/she has at home. Having calculated the sample mean number of phones, $\bar{x}$, Max finds, for each student in the sample, the squared deviation, $(x - \bar{x})^2$. The squared deviations are listed below.

$(x - \bar{x})^2$
3.24
3.24
17.64
0.04
0.64

What is the standard deviation of the number of phones for this sample of five students?

(A) 2.23
(B) 2.49
(C) 4.96
(D) 6.20
(E) 7.24

Answer

31. Let X be the score when a fair six-sided die is rolled. You are given that $\mu_X = 3.5$ and $\sigma_X = 1.708$.

A fair six-sided die is rolled 50 times. Which of the following most closely approximates the probability that the total score is at least 200?

(A) 0.000
(B) 0.019
(C) 0.385
(D) 0.483
(E) 0.500

Answer

32. The owner of a machine that makes ball bearings wishes to estimate the mean weight of all ball bearings made by the machine. The standard deviation of the weights of all ball bearings made by the machine is known. The owner at first intends to use a 95% confidence interval based on a sample of 50 ball bearings, but then decides to use a 95% confidence interval based on a sample of 200 ball bearings. Which of the following is true?

(A) The width of the second confidence interval will be 4 times the width of the first confidence interval.
(B) The width of the second confidence interval will be twice the width of the first confidence interval.
(C) The width of the second confidence interval will be half the width of the first confidence interval.
(D) The width of the second confidence interval will be a quarter of the width of the first confidence interval.
(E) The width of the second confidence interval will be one sixteenth of the width of the first confidence interval.

Answer

33. For a group of 15 male students the mean height is 70.2 inches and the mean weight is 160.3 pounds. The slope of the least squares regression line for predicting weights from heights is 2.75. What does the regression line predict for the weight of a student whose height is 68 inches?

(A) 154.25 pounds
(B) 159.5 pounds
(C) 161.1 pounds
(D) 166.35 pounds
(E) 187 pounds

Answer

34. A random sample of 25 students at a large college is taken. The IQ of each student is measured by two different methods, Method A and Method B. A regression analysis is performed, and part of the computer output is shown below.

Dependent variable: MethodB

Predictor	Coef	SE Coef	T	P
Constant	74.949887	20.88781	3.59	0.0016
MethodA	0.3409875	0.195475	1.74	0.0944

A test of the following hypotheses is performed:

H_0: There is no correlation between the results of Method A and Method B (true slope $= 0$)

H_a: There is a correlation between the results of Method A and Method B (true slope $\neq 0$)

Assuming that conditions for inference are met and using a 5% significance level, which of the following is a correct conclusion to the test?

(A) We have sufficient evidence to conclude that there is a nonzero correlation between the results of the two methods.
(B) We do not have sufficient evidence to conclude that there is a nonzero correlation between the results of the two methods.
(C) We have sufficient evidence to conclude that there is no correlation between the results of the two methods.
(D) We either do or do not have sufficient evidence to conclude that there is a nonzero correlation between the results of the two methods, according to which part of the table we use.
(E) No information regarding correlation can be inferred from the computer output.

Answer

35. Which of the following distributions is NOT symmetrical?

(A) The chi-square distribution with 8 degrees of freedom
(B) The t-distribution with 8 degrees of freedom
(C) The normal distribution with mean 52 and standard deviation 6
(D) The binomial distribution with $n = 43$ and $p = 0.5$
(E) The distribution of the score when a fair six-sided die with faces numbered 1 through 6 is rolled

Answer

36. An experiment is performed to measure the effectiveness of a new drug designed to encourage weight loss. The drug is in the form of a tablet. Forty volunteer overweight adults are randomly assigned to two groups, A and B, each with 20 people. The people in Group A are told to take the drug on a regular basis, and otherwise to go about their lives in the normal way. The people in Group B are simply told to go about their lives in the normal way, and are given no medication. After a month, the weight loss for each experimental subject is recorded. (If someone has gained weight then the weight loss is recorded as a negative number.) Throughout the experiment, each experimental subject is completely unaware of the other experimental subjects.

It is found that the people in Group A undergo a significantly greater weight loss on average than the people in Group B.

According to one of the experimenters, this result gives evidence that the chemical in the drug is effective in encouraging weight loss. Of the following, which provides the strongest argument against this claim?

(A) Twenty is not a large enough number of people to give evidence of the effectiveness of the drug.
(B) The people in Group A might have just been eating less than the people in Group B.
(C) The weight loss of the people in Group A could be attributed to the psychological factor of taking a tablet.
(D) The loss of weight could have been brought about by the warmer weather during the month of the experiment and the fact that this encourages increased exercise.
(E) Some of the people in Group A might not have taken the tablets.

Answer

37. Independent samples from two populations are taken, and a t-statistic is used to test the null hypothesis $H_0: \mu_1 = \mu_2$ against the alternative $H_a: \mu_1 > \mu_2$. The resulting p-value is 0.043. Using the same samples, a (two-sided) confidence interval will now be constructed for $\mu_1 - \mu_2$. Of the following, which is the smallest confidence level for which the confidence interval will contain zero?

(A) 90% confidence
(B) 92% confidence
(C) 94% confidence
(D) 96% confidence
(E) 98% confidence

Answer

38. In a listing of 42 houses available for rent, for each house the size (in hundreds of square feet) and the monthly rent (in dollars) is given. A scatterplot is constructed with house size on the horizontal axis and monthly rent on the vertical axis, and a linear regression analysis is run. The computer output is shown below.

Dependent variable: Rent

Predictor	Coef	SE Coef	T	P
Constant	602.50858	88.29774	6.82	<.0001
Size	24.926936	3.686165	6.76	<.0001

S = 165.62 R-sq = 53.3% R-sq (adj) = 52.2%

The size of one of the houses (in hundreds of square feet) is 32, and the monthly rent is \$1450. Which of the following (in dollars) is closest to the residual for this house?

(A) −90
(B) −70
(C) −50
(D) 50
(E) 70

Answer

39. In a large country, the heights of the men are normally distributed with mean 70 inches and standard deviation 3 inches. A recruitment agency considers a man suitable for physical labor if his height is between 67 inches and 76 inches. A man is chosen at random from the set of all men who satisfy this condition. What is the probability that this man is less than 70 inches tall?

(A) 0.341
(B) 0.417
(C) 0.5
(D) 0.683
(E) 0.715

Answer

40. A spinner has four sectors, labeled 1, 2, 3, and 4, respectively. Each spin results in one of the four sectors being selected. The spinner is spun 120 times, and the number of 1's is denoted by f_1, the number of 2's is denoted by f_2, the number of 3's is denoted by f_3, and the number of 4's is denoted by f_4.

Let p be the probability that

$$\frac{(f_1-30)^2}{30}+\frac{(f_2-30)^2}{30}+\frac{(f_3-30)^2}{30}+\frac{(f_4-30)^2}{30}$$

is greater than 6.25.

Given that the four outcomes on the spinner are equally likely, which of the following is closest to the value of p?

(A) 0.1
(B) 0.2
(C) 0.3
(D) 0.4
(E) 0.5

Answer

Sample Examination Five

SECTION I
Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A package delivery company keeps track of the weight of each package it handles. During a particular month, the company handles 14,532 packages. The weights of these packages are summarized in the following table:

Weight (pounds)	0 to < 5	5 to < 10	10 to < 20	20 to < 40	40 to < 60	60 to < 90	90 to < 120
Number of packages	3550	4215	2662	2219	887	555	444

Which of the following could be the 70th percentile package weight (in pounds) for that month?

(A) 6
(B) 19
(C) 42
(D) 48
(E) 84

Answer

2. Suppose that you are given a set of points, each with an x-coordinate and a y-coordinate. The points have been plotted on a graph. We can consider x to be the explanatory variable and y to be the response variable. The least squares regression line of y on x is the line that minimizes

   (A) the sum of the horizontal distances of the points from the line
   (B) the sum of the vertical distances of the points from the line
   (C) the sum of the squares of the horizontal distances of the points from the line
   (D) the sum of the squares of the vertical distances of the points from the line
   (E) the sum of the squares of the perpendicular distances of the points from the line

Answer

3. A random number generator produces positive whole numbers between 1 and 5 inclusive. The five possible outcomes are equally likely. The random number generator will now be used to produce ten numbers. What is the probability that exactly four of these numbers will be less than 3?

   (A) $\binom{10}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^6$
   (B) $4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^6$
   (C) $\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^6$
   (D) $\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)$
   (E) $\left(\frac{2}{5}\right)^4$

Answer

4. The board of directors of a health club is considering making changes to the club's facilities. As part of the decision making process the board wishes to find out whether members of the club feel that improvements are necessary. The board asks the manager of the club to investigate, and the manager sends out a survey that asks the question, "Do you want to spend at least $10 million to renovate the facilities?" The survey is sent to all 820 members of the club, and 808 of them respond. Seventy-five percent of the respondents answer "No," and so the manager reports to the board that a majority of the membership is against making improvements.

   Of the following, which is the most likely source of bias in the study?

   (A) Some people included in the survey did not respond.
   (B) Some parts of the population of interest were not included in the study.
   (C) The question is worded in a way that is likely to influence the response.
   (D) The study makes use of a census, and so no sampling is involved.
   (E) The study does not make use of a stratified sample.

Answer

5. There are six students enrolled in an evening class. It has been estimated that, for any evening when the class meets, the probability distribution of the number of students who attend is as shown below.

Number of Students	0	1	2	3	4	5	6
Probability	0.05	0.15	0.16	0.18	0.22	0.16	0.08

   Suppose that, on a particular evening, it is known that at least two students are attending. What is the probability that, on that evening, at most four students are attending?

   (A) 0.34
   (B) 0.43
   (C) 0.56
   (D) 0.70
   (E) 0.76

Answer

6. One hundred subjects are to be assigned to two groups (Group 1 and Group 2) for an experiment that requires a completely randomized design. A method of randomization needs to be chosen that results in two groups of equal size. Which of the following methods is correct?

   (A) For each subject, flip a coin. If the coin lands "heads," the subject is put into Group 1. If the coin lands "tails", the subject is put into Group 2.
   (B) List the subjects in alphabetical order by name. Starting with the first subject, flip a coin. If the coin lands "heads," the subject is put into Group 1. If the coin lands "tails", the subject is put into Group 2. Continue in the same way until one of the groups contains 50 subjects. Then put the remaining subjects into the other group.
   (C) List the subjects in alphabetical order by name. Using a table of random digits, assign a random four-digit number between 0001 and 9999 (inclusive) to each subject, ignoring four-digit numbers that have already been assigned. Those subjects with the 50 lowest four-digit numbers are assigned to Group 1. The remaining subjects are assigned to Group 2.
   (D) Put the male subjects into Group 1 and the female subjects into Group 2.
   (E) List the subjects in alphabetical order, and number the list 1 through 100. Those subjects with odd numbers are put into Group 1 and those with even numbers are put into Group 2.

Answer

7. Which of the following is NOT true?

   (A) The mean is a measure of the center of a distribution.
   (B) The median is a measure of the center of a distribution.
   (C) The third (upper) quartile is a measure of the spread of a distribution.
   (D) The standard deviation is a measure of the spread of a distribution.
   (E) The range is a measure of the spread of a distribution.

Answer

8. Suppose that professional photographers generally use cameras made by manufacturer C, manufacturer N, or some other manufacturer. A photography magazine publishes an article that claims that research has been done and that particular percentages of the population of professional photographers fall into these three categories (C, N, and Other). One of the manufacturers doubts the figures published in the article, and so conducts its own research. The company takes a random sample of professional photographers, and finds out for each photographer what make of camera he/she uses. It then performs a chi-square goodness of fit test, using the null hypothesis that the magazine's claim is correct. The resulting chi-square statistic is 4.19, and a 5% significance level is used. Concerning the p-value and the conclusion for this test, which of the following is true?

(A) $p = 0.123$. We have convincing evidence at the 5% level that the magazine's claim is incorrect.
(B) $p = 0.123$. We do not have convincing evidence at the 5% level that the magazine's claim is incorrect.
(C) $p = 0.123$. We have convincing evidence at the 5% level that the magazine's claim is correct.
(D) $p = 0.242$. We do not have convincing evidence at the 5% level that the magazine's claim is incorrect.
(E) $p = 0.242$. We have convincing evidence at the 5% level that the magazine's claim is correct.

Answer

9. The counselor at a high school took a random sample of girls and a random sample of boys at the school and asked each student whether he or she had sought out extra help from a teacher during the previous five school days. The counselor wishes to investigate whether the data obtained provide convincing evidence that there is a difference between the proportion of girls at the school who would say they have sought out extra help during the previous five days and the proportion of boys at the school who would say they have sought out extra help during the previous five days. Of the following, which can NOT be part of the process of conducting a hypothesis test to investigate the counselor's question using a 0.01 level of significance?

   (A) Checking that all four counts (the number of boys who say they have sought extra help, the number of boys who say they have not sought extra help, the number of girls who say they have sought extra help, and the number of girls who say they have not sought extra help) are large enough to justify using the inference procedure
   (B) Assuming a normal distribution for a population
   (C) Calculating a combined (pooled) sample proportion
   (D) Using a z-statistic to carry out the test
   (E) Given that the p-value is less than 0.01, rejecting the null hypothesis and concluding that there is convincing evidence of a difference between girls and boys in this regard

Answer

Questions 10-12 refer to the following scenario and computer output.

A student had access to rainfall and sunshine data for a particular location over a period of 60 years. She randomly selected a sample of 25 months from the 720 months available, and noted for each month in the sample the total rainfall (in millimeters) and the total sunshine (in hours). She then performed a regression analysis. Part of the computer output is shown below.

Dependent variable: Sunshine

Predictor	Coef	SE Coef	T	P
Constant	248.98414	48.45664	5.14	<.0001
Rainfall	-1.67023	0.654083	-2.55	0.0178

S = 57.834 R-sq = 22.1% R-sq (adj) = 18.7%

10. Which of the following is closest to the correlation between rainfall and sunshine for this data set?

(A) −0.470
(B) −0.432
(C) 0.018
(D) 0.432
(E) 0.470

Answer

11. The computer output gives "S = 57.834". This value gives us an idea of the

(A) slope of the regression line
(B) variability of the rainfall values
(C) variability of the sunshine values
(D) variability of the rainfall values and the sunshine values
(E) variability of the sunshine values from those predicted by the regression line

Answer

12. The regression analysis provided information about the regression line that can be used for predicting sunshine values from rainfall values using the data from the 25 months in the sample. The student now wishes to perform a significance test to determine whether these data provide convincing evidence that the equivalent regression line for all 720 months in the 60-year period has a slope that is not equal to zero. Which of the following is closest to the p-value for this test?

(A) 0
(B) 0.0178
(C) 0.0089
(D) 0.0356
(E) 0.187

Answer

13. The random variable X is normally distributed with mean 120 and standard deviation 20. Which of the following is equal to $P(110 \leq X \leq 130)$?

(A) $P(X \leq 110) + P(X \geq 130)$
(B) $P(X \geq 110) \cdot P(X \leq 130)$
(C) $P(X \leq 130) - P(X \geq 110)$
(D) $2 \cdot P(0 \leq X \leq 130)$
(E) $2 \cdot P(120 \leq X \leq 130)$

Answer

14. The weights of the sheep in a very large population are known to be normally distributed, however the population mean and the population standard deviation are unknown. A random sample of 20 sheep is selected from the population and the weight of each sheep in the sample is measured. The sample values are displayed by means of a dotplot, and the distribution of the sample values is observed to be roughly symmetrical and to include no outliers.

Following this, a hypothesis test will be performed concerning the mean weight of the population of sheep. What distribution should be used to find the p-value for this test?

(A) A normal distribution should be used because the population is known to be normally distributed.
(B) A normal distribution should be used because the distribution of the sample values is observed to be roughly symmetrical and to include no outliers.
(C) A t-distribution should be used because the sample standard deviation is unknown.
(D) A t-distribution should be used because the population standard deviation is unknown.
(E) A t-distribution should be used because the sample is too small for it to be known that the distribution of the sample mean is normal.

Answer

15. In place of a die, the ancient Greeks would roll an *astragalus*. Suppose that one astragalus has four possible outcomes, which we will name "King," "Steward," "Peasant," and "Thief," respectively. If "Peasant" is four times as likely as "King," "Thief" is four times as likely as "Steward," and "King" and "Steward" are equally likely, which of the following is closest to the probability that one roll of this astragalus results in "Peasant"?

(A) 0.1
(B) 0.2
(C) 0.3
(D) 0.4
(E) 0.5

Answer

16. A company is comparing two types of additive for gasoline: Additive A and Additive B. A random sample of ten brands of car is selected and, under controlled driving conditions, each brand is tested for gasoline mileage (in miles per gallon) with each of the two additives. For each brand of car, the difference (gas mileage with Additive B − gas mileage with Additive A) is calculated.

The distribution of the 10 differences is displayed by means of a boxplot and is found to be roughly symmetrical and to contain no outliers. The mean of the differences is 0.48 and the standard deviation of the differences is 0.766. Which of the following is a 95% confidence interval for the mean difference in gas mileage for the two additives over all brands of car?

(A) $0.48 \pm 1.812 \times \frac{0.766}{\sqrt{10}}$
(B) $0.48 \pm 2.228 \times \frac{0.766}{\sqrt{9}}$
(C) $0.48 \pm 2.228 \times \frac{0.766}{\sqrt{10}}$
(D) $0.48 \pm 2.262 \times \frac{0.766}{\sqrt{9}}$
(E) $0.48 \pm 2.262 \times \frac{0.766}{\sqrt{10}}$

Answer

17. The weights of the items made by a machine (Machine 1) are normally distributed with standard deviation 2.1 grams and unknown mean, μ_1. The weights of the items made by a second machine (Machine 2) are normally distributed with standard deviation 2.9 grams and unknown mean, μ_2. A random sample of 18 items made by Machine 1 is selected, and the resulting 95% confidence interval for μ_1 is found to be (14.223, 16.164). A random sample (independent of the first random sample) of 20 items made by Machine 2 is selected, and the resulting 95% confidence interval for μ_2 is found to be (16.388, 18.930). Which of the following is the 95% confidence interval for $\mu_2 - \mu_1$ calculated from these two samples?

(A) $(-3.807, -1.123)$
(B) $(1.650, 3.281)$
(C) $(0.867, 4.064)$
(D) $(1.123, 3.807)$
(E) $(2.165, 2.766)$

Answer

18. Suppose that a study has revealed a correlation of -0.898 between the time spent watching TV and verbal ability amongst a large number of kindergarten girls. The study was carried out using verbal test scores and by asking the girls' parents about the girls' TV-watching habits. Which of the following can NOT be concluded about these girls from the information given? (You may assume that the parents have been accurate in their reporting of the girls' TV-watching habits.)

(A) There is a strong linear relationship between the time spent watching TV and the test scores.
(B) Most of the variability in the test scores can be explained by a linear relationship between time spent watching TV and test scores.
(C) Generally a large amount of TV-watching was associated with low test scores.
(D) Generally a small amount of TV-watching was associated with high test scores.
(E) Watching a large amount of TV has caused low test scores.

Answer

19. A company has developed a new chemical designed to prevent sunburn, and is planning an experiment to determine whether the chemical is effective. A lotion has been produced consisting of the chemical in a liquid base, and an intern working for the company suggests the following experimental design:

    A set of volunteers will be randomly assigned to two groups: a treatment group and a control group. The experiment will be conducted at a location where there is consistent sunshine. All the experimental subjects will be given identical instructions as to when and how to sunbathe during a one-week period, and will have their skin inspected at the beginning and at the end of that period. In addition, people in the treatment group will be given instructions as to when and how to apply the lotion. People in the control group will be told not to use any protection against the sun.

    What useful information might be gained by including a control group in the way described rather than using only a treatment group in the experiment?

    (A) If the people in the treatment group seem to have been protected from burning, the results from the control group could give an idea as to whether it was the chemical being tested or just the liquid base that was providing this protection.
    (B) If the people in the treatment group seem to have been protected from burning, the results for the control group could enable the experimenters to detect whether this protection came about as a result of the placebo effect.
    (C) The result for the control group could give the experimenters information as to whether the combined effects of the chemical, the liquid base, and any possible placebo effect is providing protection against the sun.
    (D) The results for the control group could give the experimenters information as to whether the results of the experiment could be generalized to all people who might be exposed to the sun.
    (E) The results for the control group could give the experimenters information as to whether the randomization has produced groups that are roughly equal in terms of vulnerability to burning.

Answer

20. It is has been estimated that 20% of the students at a large college own bicycles. In order to determine the accuracy of this estimate a random sample of students from the college is selected. Each student in the sample is asked whether he or she owns a bicycle. Using the data obtained from the sample, a test is conducted of the null hypothesis $H_0: p = 0.2$ against the alternative hypothesis $H_a: p \neq 0.2$, where p is the proportion of all students at the college who own bicycles. Of the following, which is the best interpretation of the p-value for the test?

(A) The probability that H_0 is true
(B) The probability that H_a is true
(C) The probability that H_0 should be rejected
(D) The probability of getting a sample proportion at least as far from 0.2 as the one obtained, given that H_0 is true
(E) The probability of getting a sample proportion at least as far from 0.2 as the one obtained, given that H_a is true

Answer

21. The distribution of the taxable values of the homes in a large city is observed to be positively skewed. Roughly what percentage of the taxable values will be within two standard deviations of the mean?

(A) 68
(B) 95.4
(C) 97.7
(D) 99.7
(E) The percentage cannot be estimated from the information given.

Answer

22. A 95% confidence interval is to be constructed to estimate the proportion p of subscribers to a magazine who favor an increase in advertising in return for a lower price. In addition, a hypothesis test (using a 0.05 significance level) concerning the value of p will be performed. Initially, it is thought that the data will be obtained from a simple random sample of 1000 subscribers. Subsequently, use of a simple random sample of 2000 subscribers is considered. Which of the following is NOT likely to result from this increase in the sample size?

(A) A decrease in the width of the confidence interval
(B) A decrease in the probability of a Type I error in the hypothesis test
(C) A decrease in the probability of a Type II error in the hypothesis test
(D) An increase in the power of the hypothesis test
(E) A decrease in the standard deviation of the sampling distribution of the sample proportion

Answer

23. Consider the following three distributions:

- the standard normal distribution (written as "$N(0, 1)$")
- the t-distribution with 5 degrees of freedom (written as "t_5")
- the t-distribution with 10 degrees of freedom (written as "t_{10}")

The task here is to list these three distributions according to the values of their standard deviations, giving the distribution with the smallest standard deviation first. Which of the following is the correct list?

(A) $N(0, 1)$, t_5, t_{10}
(B) $N(0, 1)$, t_{10}, t_5
(C) t_5, $N(0, 1)$, t_{10}
(D) t_5, t_{10}, $N(0, 1)$
(E) t_{10}, t_5, $N(0, 1)$

Answer

24. A government education committee is planning a study of the reading ability of third graders in a large city. The committee intends to use cluster sampling for the study. As part of the planning process, the population of third grade children in the city will be divided into clusters. Of the following, which would be the most suitable criterion for the choice of clusters?

(A) Each cluster should be a simple random sample of third grade children from the city.
(B) Each cluster should as much as possible be representative of third graders in the city as a whole in terms of reading ability.
(C) All the clusters should be different from each other in terms of reading ability.
(D) Each cluster should represent a certain level of third grade reading ability.
(E) There should be as many clusters as possible.

Answer

25. A car dealership sells Sedans, Vans, and SUVs, only. The dealership has a downtown office and a suburban office. The numbers of vehicles in the various categories sold by the dealership during a particular year are shown in the table below.

	Sedans	Vans	SUVs
Downtown Office	977	244	610
Suburban Office	421	361	326

Suppose that a vehicle is chosen at random from the list of all the vehicles sold by the dealership during that year. Let A be the event that the vehicle is a sedan, and let B be the event that the vehicle was bought from the downtown office. Which of the following is true?

(A) The events A and B are independent and mutually exclusive.
(B) The events A and B are independent but not mutually exclusive.
(C) The events A and B are not independent, but they are mutually exclusive.
(D) The events A and B are not independent and are not mutually exclusive.
(E) It is impossible to tell from the information given whether the events A and B are mutually exclusive.

Answer

26. The mean length of a large population of crayfish is μ. A random sample of crayfish will be taken, and then a test will be performed of the null hypothesis H_0: $\mu = 7.5$ against the alternative hypothesis H_a: $\mu > 7.5$, using a significance level of $\alpha = 0.05$. If the true value of μ is actually greater than 7.5, then the power of the test is

(A) the probability that H_0 is rejected in favor of H_a
(B) the probability that H_0 is not rejected in favor of H_a
(C) the probability that H_a is rejected in favor of H_0
(D) 0.05
(E) 0.95

Answer

27. The administration of a large college is considering introducing random drug testing for students at the college. A random sample of 80 male students has been selected, and 23 of them have responded that they approve of random drug testing, with the remaining 57 responding that they do not approve. In addition, a random sample of 100 female students has been selected, and 38 of them have responded that they approve of random drug testing, with the remaining 62 responding that they do not approve. (The two samples were selected independently.) The administration now needs to estimate the proportion of all students at the college who would approve of the policy. Since there are roughly equal numbers of males and females at the college, this could be done by calculating the mean of the two sample proportions:

$$\frac{23/80 + 38/100}{2} = 0.334.$$

What is the standard error of this estimate?

(A) $\sqrt{\frac{(23/80)(57/80)}{80} + \frac{(38/100)(62/100)}{100}}$

(B) $\sqrt{\frac{1}{2}\left(\frac{(23/80)(57/80)}{80} + \frac{(38/100)(62/100)}{100}\right)}$

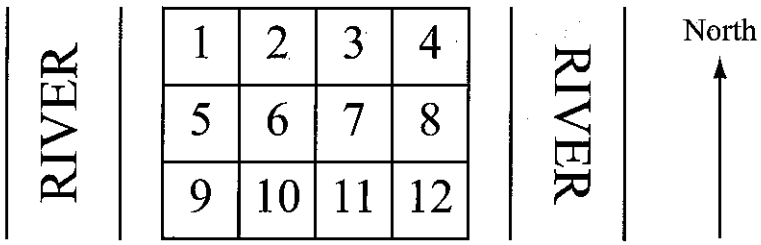
(C) $\frac{1}{2}\sqrt{\frac{(23/80)(57/80)}{80} + \frac{(38/100)(62/100)}{100}}$

(D) $\sqrt{\frac{(23/80)(57/80)}{80} - \frac{(38/100)(62/100)}{100}}$

(E) $\sqrt{\frac{1}{2}\left(\frac{(23/80)(57/80)}{80} - \frac{(38/100)(62/100)}{100}\right)}$

Answer

28. An experiment is being designed to compare the effectiveness of two types of fertilizer. Seeds of a single species of plant will be used in the experiment. The site for the experiment is a rectangular field that is positioned symmetrically between two rivers. One of the rivers is to the west of the field and the other is to the east. The field has been divided into twelve plots of approximately the same area, and the plots are numbered 1 through 12 as shown in the diagram. It is understood that the closer a plot is to a river, the more water it will receive, and the further a plot is to the south, the more light it will receive.



The designers of the experiment have decided that the plots should be blocked. Within each block, the fertilizers will be randomly assigned to equal numbers of plots.

We use the notation {1, 2, 3}, for example, to mean that plots 1, 2, and 3 form a block. Of the following blocking schemes, which is the most suitable for this experiment?

(A) {1, 3, 5, 7, 9, 11} {2, 4, 6, 8, 10, 12}
(B) {1, 2, 5, 6, 9, 10} {3, 4, 7, 8, 11, 12}
(C) {1, 2, 3, 4} {5, 6, 7, 8} {9, 10, 11, 12}
(D) {1, 5, 9} {2, 6, 10} {3, 7, 11} {4, 8, 12}
(E) {1, 4} {2, 3} {5, 8} {6, 7} {9, 12} {10, 11}

Answer

29. A random sample of size 20 is taken from a large population of horses. The heights of the horses in the sample are measured in centimeters. Using a t-distribution, the resulting 95% confidence interval for the population mean height, μ, is found to be (158.28, 160.92). Which of the following conclusions can be deduced from the information given?

(A) We have convincing evidence at the 5% level that $\mu = 158.4$.
(B) We have convincing evidence at the 5% level that $\mu \neq 158.4$.
(C) We have convincing evidence at the 2.5% level that $\mu \neq 158.4$.
(D) We do not have convincing evidence at the 5% level that $\mu \neq 158.4$.
(E) We do not have convincing evidence at the 5% level that $\mu > 158.4$.

Answer

30. An employee working at a lost-and-found office in a large railroad station is interested in the designs of umbrellas. For a sequence of 300 umbrellas that pass through the office, he notes for each its color (with categories "black," "single non-black color," and "at least two colors") and its type (with categories "telescopic" and "non-telescopic"). He compiles the results in a table, some of which is shown below. (Some of the numbers have been replaced by asterisks.)

	Telescopic	Non-Telescopic	Total
Black	*	59	*
Single non-Black Color	47	*	79
At Least Two Colors	13	*	35
Total	*	*	300

We can consider the set of 300 umbrellas to be a random sample of umbrellas that pass through the office. If color is independent of type, which of the following is closest to the expected number of black telescopic umbrellas?

(A) 70.0
(B) 78.7
(C) 79.2
(D) 115.9
(E) 127.7

Answer

31. In a large country, the distribution of the heights of married men is normal with mean 68.9 inches and standard deviation 2.9 inches. The distribution of the heights of married women is normal with mean 63.8 inches and standard deviation 2.5 inches.

A married couple is to be chosen at random. Let the random variable Z be the sum of the height of the man and the height of the woman.

Which of the following statements can be concluded from the information given?

I. $\mu_Z = 68.9 + 63.8$

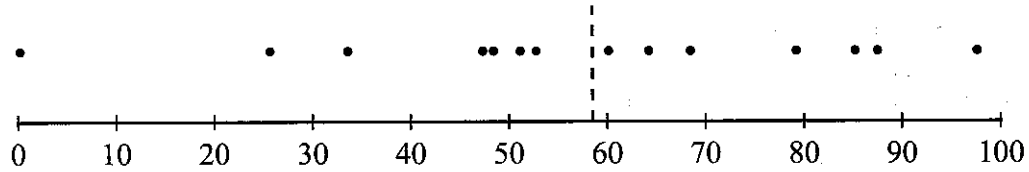
II. $\sigma_Z = \sqrt{2.9^2 + 2.5^2}$

III. The distribution of Z is normal.

(A) I only
(B) III only
(C) I and II only
(D) I, II, and III
(E) None of the statements can be concluded from the information given.

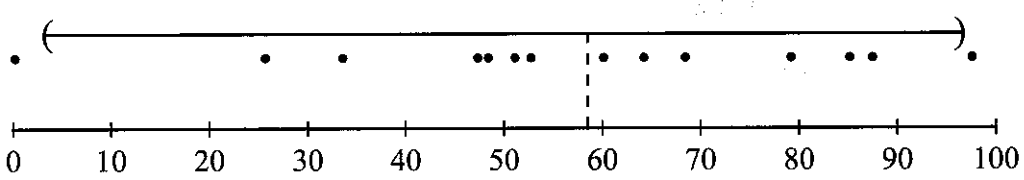
Answer

32. A random sample of size 15 is taken from a population that is known to be normally distributed. The population mean and the population standard deviation are both unknown. The sample values are displayed in the dotplot below, along with a dotted line representing the sample mean. (Note that one point cannot be seen as it is very close to another point.)

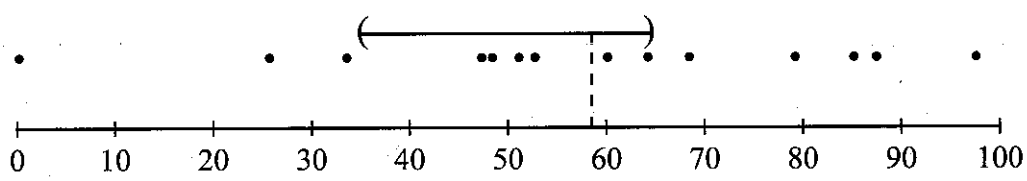


A 95% confidence interval for the population mean is calculated using the sample values. Which of the following could represent that confidence interval?

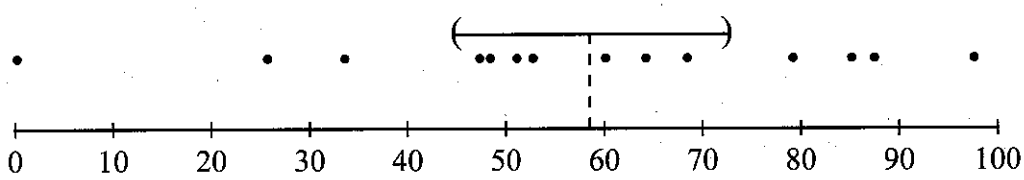
(A)



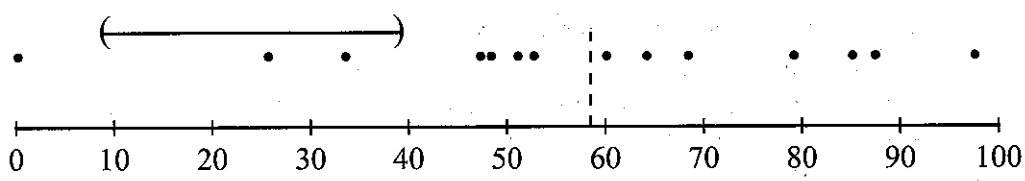
(B)



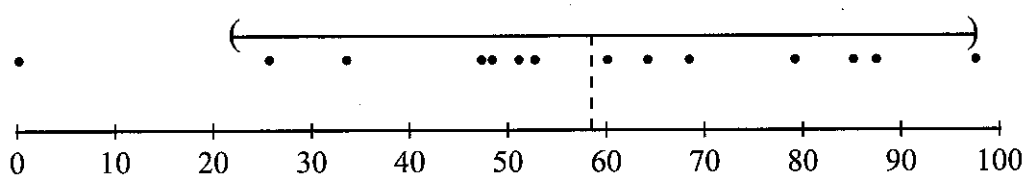
(C)



(D)



(E)



Answer

33. A data set with mean 52.3 and standard deviation 10.8 is to be transformed to a new data set using one of the transformations below, where x represents a value in the original data set and y represents its corresponding value in the new data set. If the new data set is to have mean 80.6 and standard deviation 7.9, which of the transformations is correct?

(A) $y = \frac{10.8}{7.9}x + 28.3$

(B) $y = \frac{10.8}{7.9}(x - 52.3) + 80.6$

(C) $y = \frac{7.9}{10.8}(x + 28.3)$

(D) $y = \frac{7.9}{10.8}x + 28.3$

(E) $y = \frac{7.9}{10.8}(x - 52.3) + 80.6$

Answer

34. A random sample of organisms from a particular species is obtained, and the number of males and the number of females in the sample are counted. A biologist wishes to investigate whether the data obtained provide convincing evidence that the proportion of males in the species as a whole is different from 0.5. The biologist uses a one-proportion z-test for this purpose. Which of the following inference procedures could also be used to investigate the biologist's question? (You may assume that the conditions for inference are met.)

(A) Two-sample t-test for the difference of two means
(B) Paired t-test for the difference of two means
(C) Two-sample z-test for the difference of two proportions
(D) Chi-square test for goodness of fit
(E) Chi-square test for homogeneity

Answer

35. A company employs a total of 810 people, consisting of 500 manual workers, 280 clerical workers, and 30 managerial staff. An investigator intends to conduct a survey using a sample of the company's employees. Using the company's computer records, the investigator will randomly select 50 manual workers, 28 clerical workers, and 3 managerial staff. Will the sample obtained in this way be a simple random sample of the company's employees?

(A) Yes, because every employee has an equal probability of being included in the sample.
(B) Yes, because every set of 81 employees has an equal chance of being the sample.
(C) No, because not every employee has an equal probability of being included in the sample.
(D) No, because not every set of 81 employees has an equal chance of being the sample.
(E) It is impossible to tell from the information given whether or not this will be a simple random sample of employees.

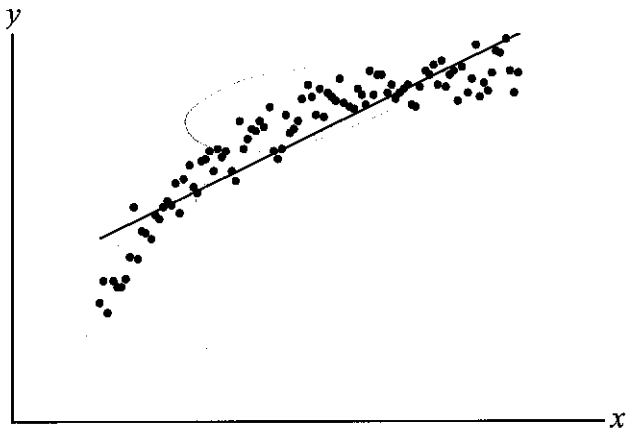
Answer

36. The standard deviation of the lifetime of a particular brand of car battery is 4.3 months. A random sample of 10 of these batteries is taken. Assuming that the lifetimes of all batteries of this brand are normally distributed, what is the probability that the mean lifetime for this sample is within 1 month of the mean lifetime for all batteries of this brand?

(A) 0.019
(B) 0.184
(C) 0.519
(D) 0.538
(E) 0.769

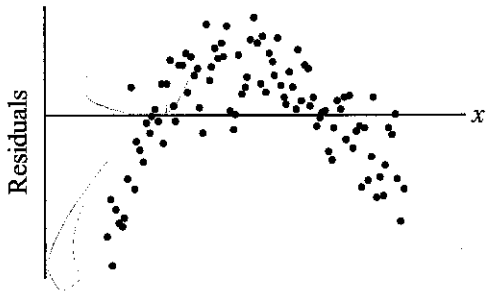
Answer

37.

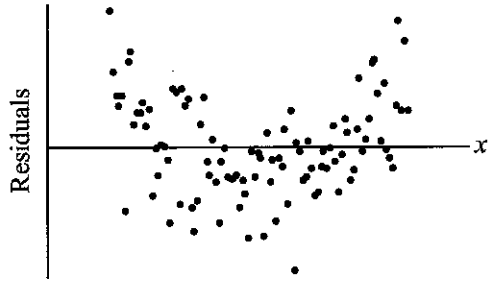


The scatterplot for a particular data set is shown above. The least squares regression line has been added. Which of the following could be the residual plot for this data set?

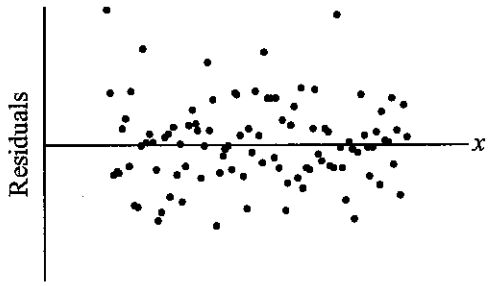
(A)



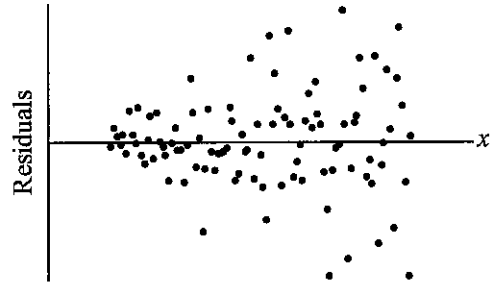
(B)



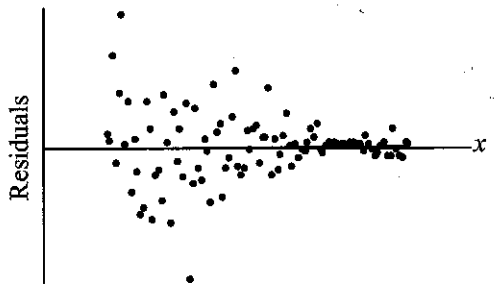
(C)



(D)



(E)



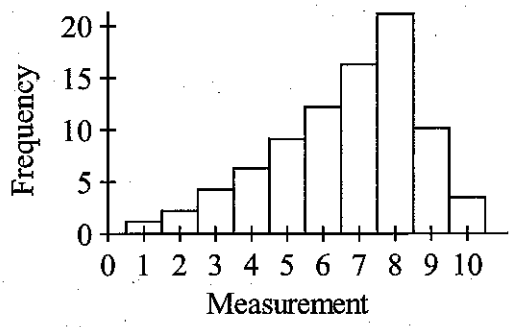
Answer

38. Drew goes to a magic store and buys a coin called "Jerry's Weighted Coin." He flips the coin 200 times and 107 of the flips result in "heads." Denoting the proportion of all flips of this coin that result in "heads" by p, which of the following can be deduced from this result?

(A) $p = 0.535$.
(B) $p = 0.5$.
(C) We are 95% confident that p is between 0.466 and 0.604.
(D) If the procedure of flipping the coin 200 times were to be repeated a large number of times, then on 95% of occasions the proportion of the 200 flips that are "heads" would be between 0.466 and 0.604.
(E) If the coin were to be flipped a further 200 times, we are 95% confident that the proportion of flips that are "heads" would be between 0.466 and 0.604.

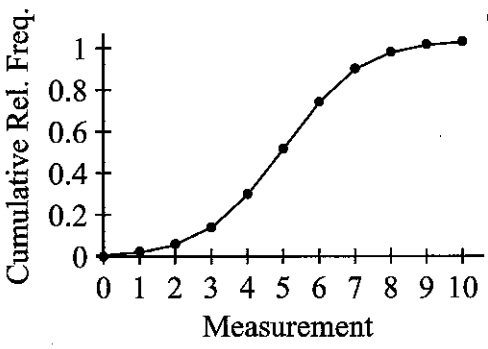
Answer

39.

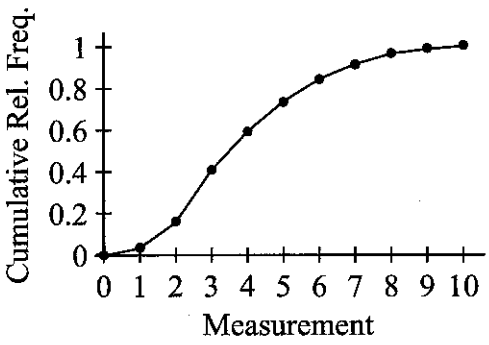


A set of measurements takes whole number values between 1 and 10 inclusive, and has the histogram shown above. Which of the following is the cumulative relative frequency graph for the same data set?

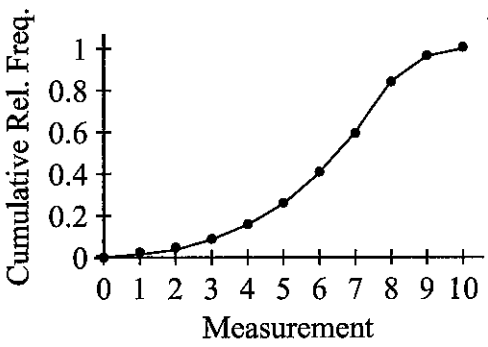
(A)



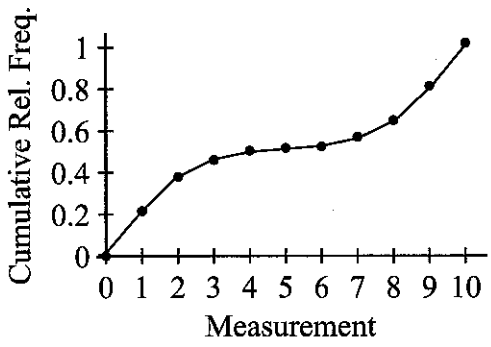
(B)



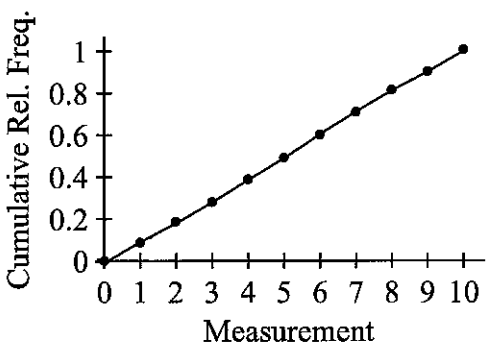
(C)



(D)



(E)



Answer

40. A bag contains 5 red beads, 3 green beads, and 2 blue beads. If three beads are picked from the bag at random (without replacement), what is the probability that the beads that are picked are all different colors?

(A) $\left(\frac{5}{10}\right)\left(\frac{3}{10}\right)\left(\frac{2}{10}\right)$

(B) $3\left(\frac{5}{10}\right)\left(\frac{3}{10}\right)\left(\frac{2}{10}\right)$

(C) $\left(\frac{5}{10}\right)\left(\frac{3}{9}\right)\left(\frac{2}{8}\right)$

(D) $3\left(\frac{5}{10}\right)\left(\frac{3}{9}\right)\left(\frac{2}{8}\right)$

(E) $6\left(\frac{5}{10}\right)\left(\frac{3}{9}\right)\left(\frac{2}{8}\right)$

Answer

Formulas and Tables

(I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1}\sum \left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\mathrm{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If $\bar{x}$ is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ, then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic ± (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\dfrac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\dfrac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Chi-square test statistic $= \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

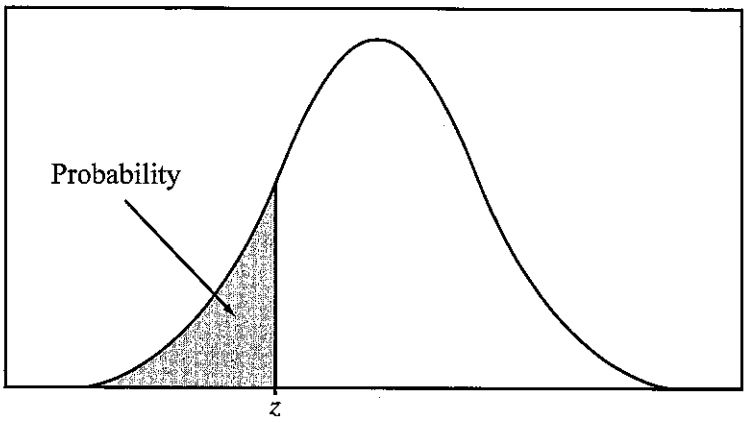


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0634	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1778	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

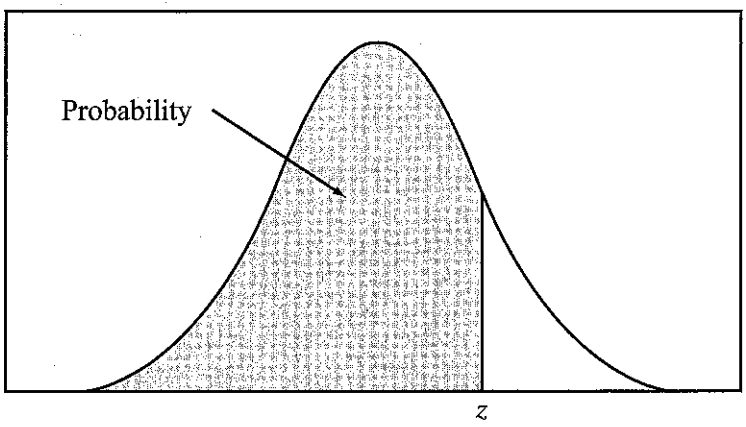


Table entry for z is the probability lying below z.

Table A (*Continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9436	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9537	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

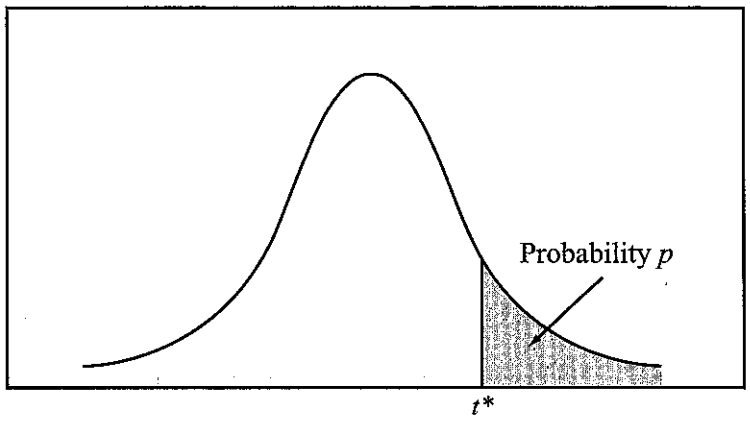


Table B t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence Level C											

Table entry for p is the point (χ^2) with the probability p lying above it.

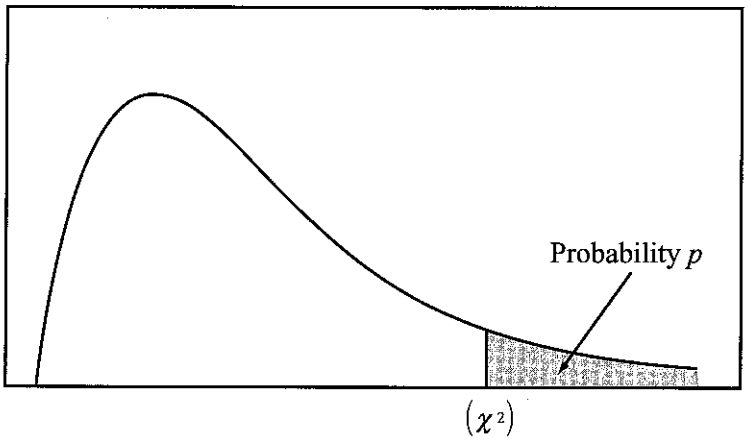


Table C critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

INDEX
(For Multiple-Choice Questions)

*Roman numerals in **bold** face type represent sample exam numbers. Other numbers are question numbers.*